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Numerical analysis of singly curved shallow composite panels under three-point bend load

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Abstract

The experimental methodology to test curved panels under three-point bend (3PB) load is assessed. The problem arises when mechanical and strength characterization of pipe material systems is required. Test specimens cut out from pipe samples oriented in the tangential direction were used to measure hoop modulus and strength. In this procedure singly curved beams with the same radius as the pipe are obtained.

The present assessment was made using three different approaches: Finite Element Method (FEM) with nonlinear geometrical analysis, curved beam theory and an approximate 2D elasticity solution. It was verified that for some cases the nonlinear effect becomes important. Furthermore the maximum span achievable, which is limited by the specimen's geometry, can prevent the use of appropriate span/thickness ratios to avoid significant shear effects.

However it was concluded that for typical GRP shell panels, with internal radius between 150 and 250 mm and a wall thickness of 12 mm, the 3PB tests display an almost linear relationship between applied load and maximum deflection. Moreover, reasonable accurate results, with an error lower than 5%, can be expected using the curved beam theory formulas. Finally some preliminarily experimental results are presented for the 250 mm radius beams with a thickness of 12 mm.

Keywords: Polymer–matrix composites (PMCs); Modeling; Orthotropic; Laminate; Shell theory

1. Introduction

The increasing use of composite materials for piping systems and other civil engineering structures has renewed interest in problems of stress–strain analysis of cylindrical multi-layered composite structures. Many analytical works about the stress analysis of composite cylindrical shells have been done during the past years. This is also related to the increase use of composite shells in several applications, such as civil engineering structures and aeronautical industry.

The interest in this type of problems is revealed by the number of articles published in recent years. Curved finite elements have been developed for composite beams [1]. Closed-form analytical solutions of in-plane static laminated curved beams of variable curvatures were derived [2]. Different analytical approaches to compute the stress–strain fields have been developed [3,4]. Closed-form 2D higher-order shear deformation theories were presented for laminated composite shells [5]. Many other research works, in this area, have been devoted to the curved sandwich beams [6–11].

The static behaviour of thin shell panels has been investigated using two-dimensional shell theories based on the Love–Kirchhoff hypotheses. Chandrashekhara and Kumar [12] presented and assessed these shell theories. The laminated shell theories provide an accurate solution for thin-walled cylinders but for thick-walled cylinders, elasticity solutions are required for an accurate determination of

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the three-dimensional stress states. Chandrashekhara and Nanjunda Rao [13] furnished an exact three-dimensional elasticity solution for an infinity transversely isotropic cylindrical shell under arbitrary discontinuous load using a displacement function approach. However this solution is based on a system of partial differential equations with variable coefficient. Chandrashekhara and Nanjunda Rao [14] used an approximation suggested by Song [15] to reduce the variable coefficient to a constant coefficient by assuming that the ratio of thickness of each ply to its middle surface radius is small and can be neglected.

The problem addressed in this work concerns the viability to characterize experimentally the mechanical properties and strength of composite pipes in the hoop direction using curved beams cut out from pipe in the tangential direction. Consequently the test samples were curved beams with the same radius as the pipe. Actually the standard used to characterize fibre-reinforced thermoplastic and thermosetting plastic composites is the ISO 14125:1998 (Fibre-reinforced plastic composites – determination of flexural properties). This standard specifies a method for determining the flexural properties of fibre-reinforced plastic composites under three-point and four-point bending. However this standard was developed for flat beams. For curved beams characterization the standard used is the ASTM D 6415/D 6415M (standard test method for measuring the curved beam strength of a fiber-reinforced polymer–matrix composite). Yet this standard is only applicable to a fiber-reinforced composite material using a 90° curved beam specimen. The curved beam must be specially manufactured for this test. It consists of two straight legs connected by a 90° bend with a 6.4 mm inner radius. Therefore a different approach was used in this work.

In Fig. 1 are depicted the two possible three-point bend (3PB) set-up configurations, concave side up or down. The applicability of thin curved beam theory formulae to calculate the hoop modulus and maximum tensile stress is assessed in this context.

The first part of the present work develops the solution for this particular case using the thin and thick curved beam theory and an approximate 2D elastic solution. Important geometric considerations for 3PB configurations related to the experimental set-up are pointed. In the second part representative cases are presented and analysed. The finite element method (FEM) was used as a reference solution, since it is a standard approach and incorporates the nonlinear geometric effects which can be important in cases where large displacement occurs. Finally comparisons are made and conclusions are drawn.

2. Curved beam theory

In Fig. 2 is depicted the typical half-view of the 3PB set-up with the general geometrical parameters, where R represents the middle plane radius.

The differential equations of equilibrium [16–18], in polar coordinates, are

$$\begin{cases} \frac{\partial}{\partial\theta}N(\theta)+Q(\theta)=0 \\ \frac{\partial}{\partial\theta}Q(\theta)-N(\theta)=0 \\ \frac{\partial}{\partial\theta}M(\theta)-RQ(\theta)=0 \end{cases} \tag{1}$$

The support reaction, V_0, is given by

$$V_0\cos(\beta)=\frac{F}{2}\Rightarrow V_0=\frac{F}{2}\sec(\beta) \tag{2}$$

Using the following boundary conditions

$$\begin{cases} Q(0)=\frac{F}{2\cos(\beta)} \\ Q(\beta)=\frac{F}{2} \\ M(0)=0 \\ N(0)=0 \end{cases} \tag{3}$$

The system is readily solved

$$\begin{cases} Q=\frac{F}{2\cos(\beta)}\cos(\theta) \\ M=\frac{FR}{2\cos(\beta)}\mathrm{sen}(\theta) \\ N=-\frac{F}{2\cos(\beta)}\mathrm{sen}(\theta) \end{cases} \tag{4}$$

For thin beams the middle surface strain ε^0 and curvature κ changes are:

$$\begin{cases} \varepsilon^0=\frac{1}{R}\left(\frac{\partial u^0}{\partial\theta}+w^0\right) \\ \kappa=\frac{1}{R^2}\left(-\frac{\partial^2 w^0}{\partial\theta^2}+\frac{\partial u^0}{\partial\theta}\right) \end{cases} \tag{5}$$

where u^0 is the tangential displacement and w^0 the radial displacement of the beam middle surface.

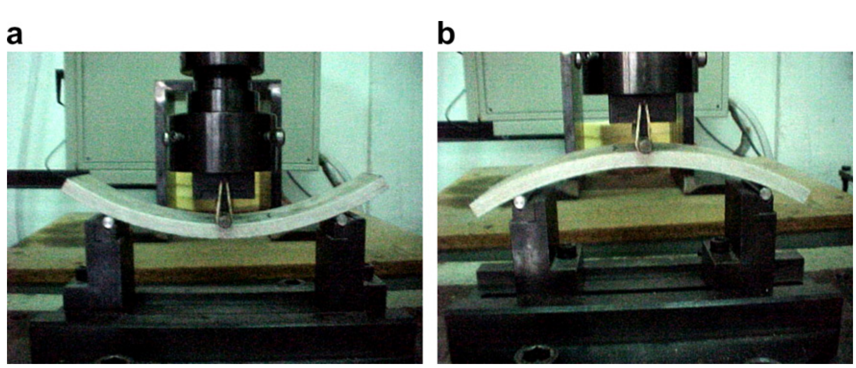

Fig. 1. Experimental 3PB set-up used to test curved composite beams with concave side (a) up and (b) down.