Composite Structures 92 (2010) 2369–2378

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Large deflection analysis of thermo-mechanical loaded annular FGM plates on nonlinear elastic foundation via DQM

O. Sepahi [a,*], M.R. Forouzan [a], P. Malekzadeh [b]

[a] *Department of Mechanical Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran*
[b] *Department of Mechanical Engineering, Persian Gulf University, Bushehr 75168, Iran*

ARTICLE INFO

Article history:
Available online 18 March 2010

Keywords:
Geometrically nonlinear
Annular FGM plates
Nonlinear elastic foundation
Thermo-mechanical loading
Differential quadrature method

ABSTRACT

The effects of three-parameter elastic foundations and thermo-mechanical loading on axisymmetric large deflection response of a simply supported annular FGM plate are investigated. An annular FGM plate, resting on a three-parameter elastic foundation under a transverse uniform loading and a transverse non-uniform temperature, is considered. The mechanical and thermal properties of the FGM plate are assumed to be graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The mathematical modeling of the plate and the resulting nonlinear governing equations of equilibrium are derived based on the first-order shear deformation theory (FSDT) in conjunction with nonlinear von Karman assumptions. A polynomial-based differential quadrature method is used as a simple but powerful numerical technique to discretize the nonlinear governing equations and to implement the boundary conditions. Finally, the effects of certain parameters, such as nonlinear foundations stiffness, volume fraction index, and temperature, on the axisymmetric large deflection response of the FGM plate are obtained and discussed in detail.

1. Introduction

Functionally graded materials have attracted the attention of many researchers since the concept of FGMs was first introduced in 1984 [1,2]. Most studies have focused on the elastic and thermo-elastic analysis of rectangular FGM plates [3–6]. Geometrically nonlinear analyses of rectangular FGM plates have also received some attention [7–12], but few works are available on circular FGM plates [13–18]. Reddy et al. [13] completed a comprehensive study of the axisymmetric bending of functionally graded circular and annular plates using the first-order shear deformation theory. Mian and Spencer [15] established a large class of exact solutions of the three dimensional elasticity equations for functionally graded and laminated elastic materials. Ma and Wang [19] studied the axisymmetric nonlinear bending and the thermal post-buckling behavior of a functionally graded circular plate under thermo-mechanical loading within the framework of the classical nonlinear von Karman plate theory. Shooting method was employed to investigate the effects of material constant, boundary conditions, and imposed temperature on the response of circular FGM plates. Li et al. [20] investigated the geometrically nonlinear post-buckling of an imperfect circular FGM plate subjected to both mechanical load and transverse non-uniform temperature rise.

Recently, Nosier and Fallah [21] presented the nonlinear analysis of functionally graded circular plates under asymmetric transverse loading, based on the first-order shear deformation plate theory with von Karman non-linearity. A perturbation technique, in conjunction with Fourier series method to model the problem asymmetries, is used to obtain the solution for various clamped and simply supported boundary conditions.

This study is devoted to the geometrically nonlinear analysis of a simply supported annular FGM plate resting on a three-parameter elastic foundation. To the best of the authors' knowledge, no study is reported in the literature on large deflection analyses of annular FGM plates resting on nonlinear foundations and problems in this area still await solution. The plate is subjected to both mechanical load and transverse non-uniform temperature rise. The mechanical and thermal properties of the FGM plate are assumed to be graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. Based on Hamilton principle, the nonlinear governing equations of equilibrium are obtained according to the first-order shear deformation theory (FSDT) in conjunction with nonlinear von Karman assumptions. The differential quadrature method is employed as a simple but accurate and fast convergent method to discretize the resulting set of nonlinear equilibrium equations and to implement the boundary conditions. Applicability and accuracy of the method were illustrated by Malekzadeh et al. who applied it to the nonlinear analysis of a composite plate [22–25].

* Corresponding author. Tel.: +98 311 391 3933; fax: +98 311 391 2628.
E-mail address: omidsepahi@me.iut.ac.ir (O. Sepahi).

0263-8223/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2010.03.011

Finally, the effects of thermal and mechanical loading parameters, various foundation parameters, temperature gradient, and volume fraction index on the axisymmetric large deflection response of a simply supported annular FGM plate are investigated and discussed in detail.

2. Problem formulation

Consider a functionally graded annular plate with an inner radius R_1, an outer radius R_2, and a uniform thickness h, as shown in Fig. 1. The mid-plane of the plate is referred to the cylindrical coordinate system (r, θ, z) in the radial and circumferential directions, where distances from the mid-plane are measured by a coordinate z, positive upward.

2.1. Mechanical and thermal properties of the FGM plate

FGMs are usually modeled as an inhomogeneous isotropic linear elastic material. Here, it is assumed that the material properties of FGM plates vary continuously through the plate thickness as a function of the volume fraction and the properties of constituent materials, from full ceramic at the top surface to full metal at the bottom. A simple power law distribution is used for the volume fraction of the constituents, metal and ceramic, as follows [19,20]:

$$V_m(z) = \left(\frac{1}{2} - \frac{z}{h}\right)^n, \quad V_c(z) = 1 - V_m(z) \tag{1}$$

where subscripts m and c refer to the metal and ceramic constituents, respectively; V_m and V_c denote the volume fraction of metal and ceramic, respectively; n is called the volume fraction index or material constant; and z is the thickness coordinate ($-h/2 \leqslant z \leqslant h/2$). Fully metal and fully ceramic are represented, respectively, by zero and infinity values of the material constant, n. Based on the linear role of the mixture, the effective mechanical and thermal properties of FGMs can be expressed as [19,20]:

$$P(z) = P_m V_m + P_c V_c \tag{2}$$

where P_m and P_c denote the specific properties of metallic and ceramic constituents, respectively. Therefore, all the mechanical and thermal properties of the FGM plate such as Young's modulus, E; thermal expansion coefficient, α; and thermal conductivity coefficient, K, can be written as

$$\begin{aligned} E(z) &= E_c + (E_m - E_c)V_m \\ \alpha(z) &= \alpha_c + (\alpha_m - \alpha_c)V_m \\ K(z) &= K_c + (K_m - K_c)V_m \end{aligned} \tag{3}$$

Poisson's ratio, ν, is assumed to be constant for simplicity and convenience.

2.2. Temperature field of the FGM plate

For our purposes, it is assumed that the temperature gradient $T(z)$ from the stress free state is governed by the well-known heat transfer equation, which varies only along the thickness direction. Therefore, by denoting the temperature at the top surface and that at the bottom by T_m and T_c, respectively, the transverse non-uniform temperature distribution $T(z)$ can be determined using the following boundary value problem [19,20]

$$\frac{d}{dz}\left(K(z)\frac{dT(z)}{dz}\right) = 0, \quad T(-h/2) = T_m, \quad T(h/2) = T_c \tag{4}$$

Temperature distribution is easy to obtain from Eq. (4) as

$$T(z) = T_m + (T_c - T_m)\int_{-h/2}^{z} K(\xi)d\xi \bigg/ \int_{-h/2}^{h/2} K(z)dz \tag{5}$$

2.3. Governing equations of the problem

Based on the first-order shear deformation plate theory (FSDT), displacement field in the cylindrical coordinate system for the present problem can be written as [13]

$$\begin{aligned} U(r,\theta,z) &= u(r) + z\varphi(r) \\ W(r,\theta,z) &= w(r) \end{aligned} \tag{6}$$

where $u(r)$ and $w(r)$ denote the displacements at the mid-plane of the plate in the r and z directions, respectively, and ϕ represents the small transverse normal rotation about the θ-axis. According to the von Karman nonlinear strain–displacement relations of elasticity, strains components are obtained as follows:

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial U}{\partial r} + \frac{1}{2}\left(\frac{\partial W}{\partial r}\right)^2 \\ \frac{U}{r} \\ \kappa_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 \\ \frac{u}{r} \\ 0 \end{Bmatrix} + z\begin{Bmatrix} \frac{\partial \varphi}{\partial r} \\ \frac{\varphi}{r} \\ \varphi + \frac{\partial w}{\partial r} \end{Bmatrix} \tag{7}$$

which in compact form can be rewritten as

$$\varepsilon = \varepsilon_0 + z\kappa \tag{8}$$

where

$$\begin{Bmatrix} \varepsilon_r^0 \\ \varepsilon_\theta^0 \\ \gamma_{rz}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 \\ \frac{u}{r} \\ 0 \end{Bmatrix} \begin{Bmatrix} \kappa_r \\ \kappa_\theta \\ \kappa_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \varphi}{\partial r} \\ \frac{\varphi}{r} \\ \varphi + \frac{\partial w}{\partial r} \end{Bmatrix} \tag{9}$$

The constitutive relations for the plane stress state, based on Hook's law, are written as

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{r\theta} \end{Bmatrix} - \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \alpha T \right) \tag{10}$$

where

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}, \quad Q_{12} = \nu Q_{11}, \quad Q_{66} = \frac{E}{2(1+\nu)} \tag{11}$$

and T is the temperature rise from a stress free state. Note that the effective properties of the plate vary through the thickness according to Eq. (2) and that the elastic coefficient Q_{ij} is, thus, a function of z. The resultant forces, moments, and transverse force of the stresses are defined by

$$\begin{aligned} (N_r, N_\theta) &= \int_{-h/2}^{h/2} (\sigma_r, \sigma_\theta) dz \\ (M_r, M_\theta) &= \int_{-h/2}^{h/2} (\sigma_r, \sigma_\theta) z dz \\ Q_r &= \int_{-h/2}^{h/2} \tau_{rz} dz \end{aligned} \tag{12}$$

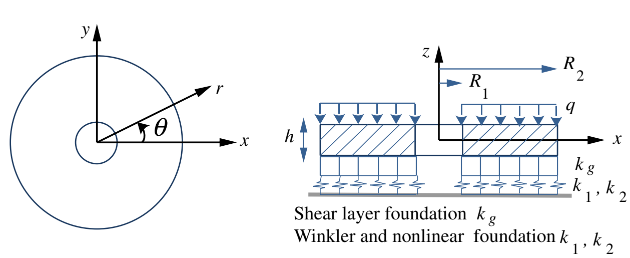

Fig. 1. The geometry of annular FGM plate.