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Three-dimensional analysis for thermoelastic response of functionally graded fiber reinforced cylindrical panel

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ABSTRACT

In this paper three-dimensional steady-state response of a functionally graded fiber reinforced cylindrical panel are studied. The functionally graded orthotropic panel is simply supported at the edges and assumed to have a linear variation of reinforcement volume fraction in the radial direction. Suitable temperature and displacement functions that identically satisfy the simply supported boundary conditions are used to reduce the thermoelastic equilibrium equations to a set of coupled ordinary differential equations with variable coefficients, which can be solved by differential quadrature method. Results are presented for the fiber reinforced functionally graded cylindrical panel with graded fiber volume fractions and compared with traditional discretely laminated composite panel. Results indicate the advantages of using functionally graded fiber reinforced composite shell with graded fiber volume fractions over traditional discretely laminated composite shells.

1. Introduction

Functionally graded materials (FGM) are heterogeneous materials in which the elastic and thermal properties changes from one surface to the other, gradually and continuously. The material is constructed by smoothly changing the volume fraction of its constituent materials. FGMs offer great promise in applications where the operating conditions are severe, including spacecraft heat shields, heat exchanger tubes, plasma facings for fusion reactors, engine components and high power electrical contacts or even magnets. For example, in a conventional thermal barrier coating for high temperature applications, a discrete layer of ceramic material is bonded to a metallic structure. However, the abrupt transition in material properties across the interface between distinct materials can cause large interlaminar stresses and lead to plastic deformation or cracking [1]. These adverse effects can be alleviated by functionally grading the material to have a smooth spatial variation of material composition.

Classical method of analysis is to combine the equilibrium equations with the stress–strain and strain–displacement relations to arrive at the governing equations in terms of the displacement components, called the Navier equations. A method to solve the Navier equations is through potential function method. The method is used for isotropic materials, as well as functionally graded materials. Zhang et al. [2] presented an analytical procedure for functionally gradient material cylinder with axial symmetry, based on thermal elasticity theory. Liew et al. [3] studied the analysis of the thermo-mechanical behavior of hollow circular cylinders of functionally graded material. Tutuncu and Ozturk [4] presented the closed-form solutions for stresses and displacements in functionally graded cylindrical and spherical vessels subjected to internal pressure alone, using the infinitesimal theory of elasticity. Pelletier Jacob et al. [5] presented generalized plane strain thermal stress of a functionally graded thick cylindrical shell subjected to thermal and mechanical loads. Noda [6] presented an explicit solution composed of two thermoelastic potential functions in cylindrical coordinates for steady-state, axially asymmetric thermal stresses in an isotropic long hollow cylinder. Misra and Achari [7] studied the exact solution of thermal stresses in a hollow anisotropic cylinder of finite length arising from axisymmetric temperature variations at the plane ends, the inner and outer curved surfaces being in contact with rigid and smooth insulators, using potential functions of displacement. Katsuo et al. [8] studied the thermal stress distributions of laminated composite finite hollow cylinders restricted at both ends and at one end of the cylinders, using the thermoelastic displacement potential and Michell's stress functions based on the axisymmetrical theory of elasticity. Chen [9] analyzed the problems of linear thermoelasticity in a transversely isotropic hollow cylinder of finite length by a direct power series approximation through the application of the Lanczos–Chebyshev method. Jabbari et al. [10] studied the mechanical and thermal stresses in a functionally graded hollow cylinder due to radially symmetric loads. The exact solution of steady-state two-dimensional axisymmetric mechanical and thermal stresses for a short hollow cylinder made of functionally graded material

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was presented by Jabbari et al. [11]. Bahtui and Eslami [12,13] studied the coupled thermoelastic response of a functionally graded circular cylindrical shell. The coupled thermoelastic and the energy equations were simultaneously solved for a functionally graded axisymmetric cylindrical shell subjected to thermal shock load. Ootao and Tanigawa [14] presented the transient thermoelastic problem of functionally graded thick strip due to non-uniform heat supply. They obtained the exact solution for the two-dimensional temperature change in a transient state and thermal stresses of a simple supported strip under the state of plane strain. Jabbari et al. [15] presented the axisymmetric mechanical and thermal stresses in thick long FGM cylinders. They presented a direct method of solution of Navier equations, rather than potential functions method. The heat conduction and Navier equations were solved analytically, using the generalized Bessel function and complex form of Fourier integral. Shahsiah and Eslami [16] studied thermal buckling of functionally graded cylindrical shell by using shell theories and finite element formulation. Kordkheili et al. [17] presented thermoelastic analysis of functionally graded cylinders under thermal loading. They obtained the governing equations based on the minimum potential energy and then solved by using power series. Thermoelastic and vibration analysis of functionally graded cylindrical shells were studied by Zhao et al. [18]. The functionally graded was under thermo-mechanical load.

In this paper three-dimensional analysis for thermoelastic response of a functionally graded fiber reinforced panel is studied. The results are presented for an orthotropic panel with linear continuous grading of volume fractions through the thickness and compared with conventional discretely laminated cylindrical panels.

2. Problem formulation

An FGM cylindrical panel with its co-ordinate system (r, θ, z) is shown in Fig. 1, where r, θ, z are in the radial, circumferential and axial directions of the panel. The effective mechanical properties of the fiber reinforced cylindrical panel are obtained based on a micromechanical model as follows [19,20]:

$$E_{11} = V_f E_{11}^f + V_m E_{11}^m \tag{1}$$

$$\frac{1}{E_{ii}} = \frac{V_f}{E_{ii}^f} + \frac{V_m}{E_{ii}^m} - V_f V_m \frac{\upsilon_f^2 E_{ii}^m / E_{ii}^f + \upsilon_m^2 E_{ii}^f / E_{ii}^m - 2\upsilon^f \upsilon^m}{V_f E_{ii}^f + V_m E_{ii}^m} \quad i = 2, 3 \tag{2}$$

$$\frac{1}{G_{ij}} = \frac{V_f}{G_{ij}^f} + \frac{V_m}{G_{ij}^m} \quad ij = 12, 23 \text{ and } 13 \tag{3}$$

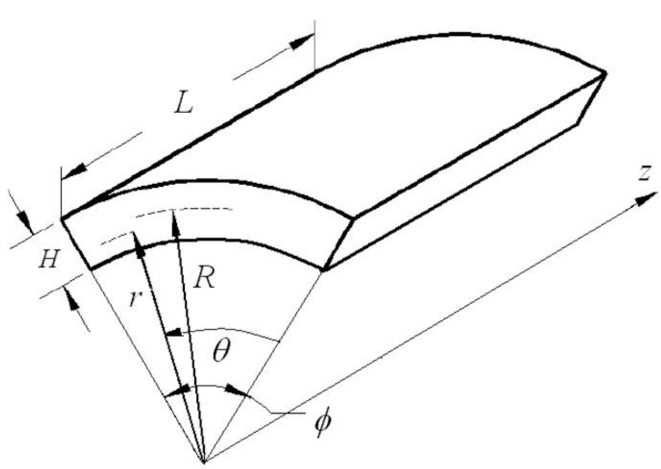

Fig. 1. Geometry of a cylindrical panel.

$$\upsilon_{ij} = V_f \upsilon^f + V_m \upsilon^m \quad ij = 12, 23 \text{ and } 13 \tag{4}$$

$$\alpha_{11} = \frac{V_f E_{11}^f \alpha_{11}^f + V_m E_{11}^m \alpha_{11}^m}{V_f E_{11}^f + V_m E_{11}^m} \tag{5}$$

$$\alpha_{ii} = (1 + \upsilon^f) V_f \alpha_{ii}^f + (1 + \upsilon^m) V_m \alpha_{ii}^m - \upsilon_{1i} \alpha_{11} \quad i = 2, 3 \tag{6}$$

$$k_{11} = V_f k_{11}^f + V_m k_{11}^m \tag{7}$$

$$\frac{1}{k_{ii}} = \frac{V_f}{k_{ii}^f} + \frac{V_m}{k_{ii}^m} \quad i = 2, 3 \tag{8}$$

where E_{ii}, G_{ij}, υ_{ij}, α_{ii}, k_{ii}, V_f and V_m are elasticity modulus, shear modulus, Poisson's ratio, thermal expansion coefficient, thermal conductivity, fiber volume fraction and matrix volume fraction. For orthotropic cylindrical panel, the following variation of the reinforcement volume fraction is assumed [5]:

$$V = V_i + (V_o - V_i)\left(\frac{r - r_i}{r_o - r_i}\right)^P \tag{9}$$

where V_i and V_o which have values that range from 0 to 1, denote the value fractions on the inner and outer surfaces, respectively. The exponent "P" controls the volume fraction profile through the shell's thickness which is set to 1 for numerical results.

The mechanical constitutive relations, which relates the stresses to the strains are as follows:

$$\begin{Bmatrix} \sigma_z \\ \sigma_\theta \\ \sigma_r \\ \tau_{r\theta} \\ \tau_{zr} \\ \tau_{z\theta} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_r \\ \gamma_{r\theta} \\ \gamma_{zr}\gamma_{z\theta} \end{Bmatrix} - \begin{bmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{33} \\ 0 \\ 0 \\ 0 \end{bmatrix} T \tag{10}$$

where C_{ij} are elastic stiffnesses and β_{ij} are the stresses module that are related to the thermal expansion coefficients α_{ii} as follows:

$$\begin{Bmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{33} \end{Bmatrix} = \begin{Bmatrix} C_{11}\alpha_{11} + C_{12}\alpha_{22} + C_{13}\alpha_{33} \\ C_{12}\alpha_{11} + C_{22}\alpha_{22} + C_{23}\alpha_{33} \\ C_{13}\alpha_{11} + C_{23}\alpha_{22} + C_{33}\alpha_{33} \end{Bmatrix} \tag{11}$$

Since the shell is graded in the radial direction, the material properties k_{ii}, C_{ij} and α_{ii} are functions of the radial coordinate.

In the absence of body forces, the governing equations are as follows:

$$\begin{aligned} &\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{z\theta}}{r\partial \theta} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0 \\ &\frac{\partial \tau_{\theta z}}{\partial z} + \frac{\partial \sigma_\theta}{r\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} = 0 \\ &\frac{\partial \tau_{zr}}{\partial z} + \frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \end{aligned} \tag{12}$$

The three-dimensional steady-state heat conduction equation, expressed in terms of the local coordinates, is:

$$\frac{\partial q_r}{\partial r} + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z} + \frac{q_r}{r} = 0 \tag{13}$$

where q_θ, q_r and q_z are the components of the heat flux vector. Fourier's law of heat conduction, which relates the heat flux to the temperature gradient, is:

$$\begin{Bmatrix} q_z \\ q_\theta \\ q_r \end{Bmatrix} = -\begin{Bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{Bmatrix} \begin{Bmatrix} \frac{\partial T}{\partial z} \\ \frac{1}{r}\frac{\partial T}{\partial \theta} \\ \frac{\partial T}{\partial r} \end{Bmatrix} \tag{14}$$

where T is the change in temperature of a material particle from that in the stress-free reference configuration and k_{ii} are the