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Three-dimensional piezoelasticity solution for dynamics of cross-ply cylindrical shells integrated with piezoelectric fiber reinforced composite actuators and sensors

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A B S T R A C T

A benchmark three-dimensional (3D) exact piezoelasticity solution is presented for free vibration and steady state forced response of simply supported smart cross-ply circular cylindrical shells of revolutions and panels integrated with surface-bonded or embedded monolithic piezoelectric or piezoelectric fiber reinforced composite (PFRC) layers. The effective properties of PFRC laminas for the 3D case are obtained based on a fully coupled iso-field model. The governing partial differential equations are reduced to ordinary differential equations in the thickness coordinate by expanding all entities for each layer in double Fourier series in span coordinates, which identically satisfy the boundary conditions at the simply-supported ends. These equations with variable coefficients are solved using the modified Frobenius method, wherein the solution is constructed as a product of an exponential function and a power series. The unknown constants of the general solution are finally obtained by employing the transfer matrix method across the layers. Results for natural frequencies and the forced response are presented for single layer piezoelectric and multilayered hybrid composite and sandwich shells of revolution and shell panels integrated with monolithic piezoelectric and PFRC actuator/sensor layers. The present benchmark solution would help assess 2D shell theories for dynamic response of hybrid cylindrical shells.

1. Introduction

Hybrid composite and sandwich shells with embedded or surface-mounted piezoelectric sensors and actuators form a very important part of the new generation of smart structures with potential applications in aerospace vehicles, automobiles, micro air vehicles, sport products, etc. Recent developments of the piezoelectric fiber reinforced composite (PFRC) with high strength (maximum strain of 2300 microstrain), toughness, operating range (−1500 to 2800 V) and life (200 million cycles), conformability to curved shell surfaces and broadband have widened the scope of its application [1]. Due to the presence of high layerwise inhomogeneity in mechanical and electric properties in the hybrid laminates, modeling of structures of such laminates requires special attention. The most accurate global laminate level as well as local layerwise response of hybrid laminated structures can be obtained by the exact analytical solution based on the three-dimensional (3D) piezoelasticity. In these solutions, no ad hoc assumptions are made on the variations of the field variables across the thickness. These solutions provide insight into the complex electromechanical behaviour of the hybrid laminated structures and also serve as useful benchmarks for the assessment of 2D plate and shell theories.

The governing equations of 3D piezoelasticity for cylindrical shells involve variable coefficients as a function of the radial coordinate, which prevent a straight forward closed form solution unlike in case of constant coefficients. An approximate 3D elasticity solution for free vibration response of simply-supported doubly curved shallow orthotropic laminated shells was presented by Bhimaraddi [2] by dividing each layer into a number of sublayers and assuming the coefficients to be constant in each sublayer. Soldatos and his coworkers [3,4] used a similar successive approximation approach to obtain free vibration response of simply-supported isotropic and cross-ply composite circular cylindrical shells of revolution and panels. A 3D piezoelasticity solution for the free vibration of an orthotropic cylindrical shell with a monolithic piezoelectric layer bonded to its outer surface was presented by Chen and Shen [5] employing power series expansion method. In this formulation, however, the piezoelectric transverse extension and shear strain coefficients (e_{33}, e_{15}, e_{25}) are taken as zero and the piezoelectric constants considered for radial poling direction are not consistent with the elastic constants. Chen et al. [6] have presented an approximate 3D piezoelasticity solution for the static and free vibration response of cross-ply laminated piezoelectric finite circular cylindrical panels with imperfect interfacial bonding by using the successive approximation method of Bhimaraddi [2]. In this work, however, the poling of the monolithic piezoelectric layers and the applied electric field are considered to be along the axial direction, which is unrealistic and ineffective for sensing

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and actuation in structures. For such applications, the poling of the piezoelectric layers and the electric field should be along the thickness (radial) direction. Exact 3D piezoelasticity solutions for finite simply-supported cross-ply cylindrical shells and panels with radially polarized piezoelectric layers have been presented for static response by Xu and Noor [7], and Kapuria et al. [8,9]. They employed the modified Frobenius method [10] for the solution, which yields faster convergence than the conventional Frobenius method and the power series method [11]. This method involves no approximations of the kind in Refs. [2–4,6].

For large scale structural control applications such as in aerospace structures, monolithic piezoelectric actuators and sensors suffer from certain shortcomings with regard to tailorable anisotropic actuation i.e. directional actuation, robustness against damage during use and handling, ability to cover the entire structure for distributed actuation and sensing, and conformability to curved structural members such as shells and tubes. Piezoelectric fiber reinforced composites have been introduced to address these concerns. A number of micromechanics models that have been proposed for the determination of effective properties of piezoelectric composites based on Voigt type iso-field [12,13], Mori Tanaka [14], self-consistent [14,15] and asymptotic homogenization [16,17] methods, consider poling and electric field directions parallel to the fiber axis, causing d_{33} effect. Which such composites with piezo-fibers oriented along the thickness direction (Fig. 1a) are applied in ultrasonic transducers, for structural applications, the stiff piezoceramic fibers must be oriented in the plane of the structures. Hence, the above models can not be employed in the present case. A micromechanics model for PFRC with poling and electric field directions perpendicular to the fiber (Fig. 1b) was first presented by Bent [18] using the iso-field method. In this work, even though a general methodology for calculation of effective material properties for the 3D stress field was briefly outlined, detailed closed form solutions and the results for effective material properties were presented by considering uni-axial stress fields. Mallik and Ray [19] presented a simpler model using the uniform fields concept, but in this formulation, the electric field is assumed to be the same in both piezoelectric fiber and elastic matrix, which is not achievable by applying potential difference across the lamina due to large difference in their dielectric constants. The formulation, thus, gives unrealistically high values of effective piezoelectric strain constants.

In this work, a 3D exact piezoelasticity solution is presented for the free vibration and steady state forced vibration response of finite, simply-supported cross-ply laminated circular cylindrical shells of revolution and shell panels with surface-bonded or embedded monolithic piezoelectric or PFRC sensory/actuator layers. A fully coupled micromechanical model based on iso-field assumptions is developed for computing the effective electromechanical material properties of a PFRC lamina, which are required for the 3D analysis. For the analysis of the laminate, a mixed formulation is used by expressing the governing field equations in terms of eight primary variables: displacements u, v, w, transverse stress $\sigma_r, \tau_{zr}, \tau_{r\theta}$, electric potential ϕ and transverse electric displacement D_z. The basic entities for each layer along with the prescribed electromechanical loading functions are expanded in double Fourier series in the span coordinates to satisfy the boundary conditions at the simply-supported ends. The governing equations reduce to a system of eight ordinary differential equations in the radial coordinate with variable coefficients. The solution is constructed using the modified Frobenius method as a product of an exponential function and a power series. This expansion is substituted in the governing equations, and the coefficients of the terms of different degree are set to zero. This yields a linear eigenvalue problem for the exponent of the exponential term with eight eigenvalues and a set of recursive relations for the coefficients of the power series. The eight arbitrary constants in general solution for each layer are determined from the boundary (top and bottom surfaces) and interface conditions. Since the piezoelasticity equations are used, the solution presented is valid for thin, thick, shallow as well as deep shells. Numerical results are presented for the natural frequencies and steady state harmonic response of cross-ply single layer piezoelectric and PFRC shells, and hybrid composite and sandwich shells with monolithic piezoelectric and PFRC layers. Results are presented for both shells of revolution and shell panels. The benchmark steady state solutions for harmonic loads are well suited to assess the accuracy and the domain of validity of 2D hybrid shell theories.

2. Effective material properties of PFRC materials

The effective electroelastic constants of PFRC materials are determined from the properties of individual phases (fiber and matrix) by using the iso-field approach generalized for electromechanical field by Bent [18]. In order to have a unified treatment, both fiber and matrix are assumed to be piezoelectric materials, which are of orthotropic class mm2 symmetry, with principal material axes x_1, x_2, x_3 and polarized along the thickness direction x_3. The 3D linear constitutive equations of such a piezoelectric continuum are given by [20]

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \\ D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 & 0 & 0 & d_{31} \\ s_{12} & s_{22} & s_{23} & 0 & 0 & 0 & 0 & 0 & d_{32} \\ s_{13} & s_{23} & s_{33} & 0 & 0 & 0 & 0 & 0 & d_{33} \\ 0 & 0 & 0 & s_{44} & 0 & 0 & 0 & d_{24} & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 & d_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{15} & 0 & \epsilon_{11} & 0 & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 & 0 & \epsilon_{22} & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 & 0 & 0 & \epsilon_{33} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \\ E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (1)$$

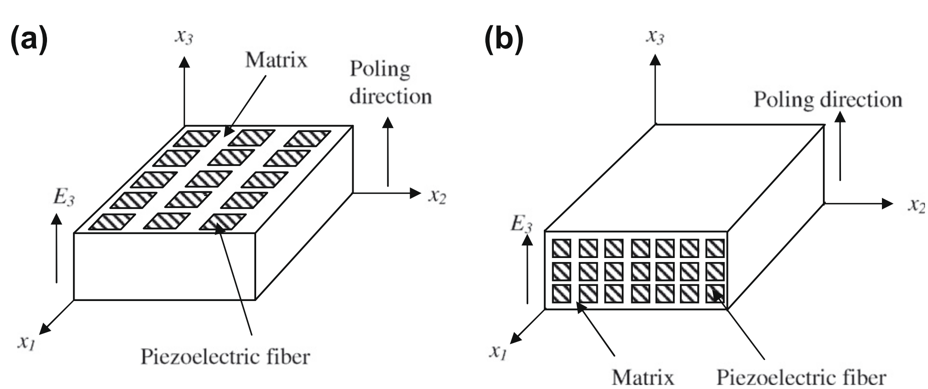

Fig. 1. Schematic representation of PFRC with poling and electric field directions (a) parallel and (b) normal to fiber direction.