Composite Structures 92 (2010) 2497–2511

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A mixed Ritz-DQ method for forced vibration of functionally graded beams carrying moving loads

S.M.R. Khalili [a,b,*], A.A. Jafari [a], S.A. Eftekhari [a]

[a] Centre of Excellence for Research in Advanced Materials and Structures, Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Tehran, Iran
[b] Faculty of Engineering, Kingston University, London, UK

ARTICLE INFO

Article history:
Available online 26 February 2010

Keywords:
DQM
Rayleigh–Ritz method
FG beam
Moving load
Inertia effect
Forced vibration

ABSTRACT

A mixed method is presented to study the dynamic behavior of functionally graded (FG) beams subjected to moving loads. The theoretical formulations are based on Euler–Bernoulli beam theory, and the governing equations of motion of the system are derived using the Lagrange equations. The Rayleigh–Ritz method is employed to discretize the spatial partial derivatives and a step-by-step differential quadrature method (DQM) is used for the discretization of temporal derivatives. It is shown that the proposed mixed method is very efficient and reliable. Also, compared to the single-step methods such as the Newmark and Wilson methods, the DQM gives better accuracy using larger time step sizes for the cases considered. Moreover, effects of material properties of the FG beam and inertia of the moving load on the dynamic behavior of the system are investigated and analyzed.

1. Introduction

The study of behavior of functionally graded materials (FGMs) has been an interesting topic of considerable focus during the past decades. The intensity and rapid growth of research into this class of materials is actually due to their continuously varying material properties, which gives great advantages over the conventional homogeneous and layered materials. For example, FGMs consisting of metallic and ceramic components used in dynamic systems can improve both the mechanical and thermo-mechanical performances of the system. On the other hand, the cracking and delamination phenomenon which are often observed in conventional multi-layered systems are avoided due to the smooth transition between the properties of the components in FGMs [1–3]. Therefore, it is of great importance to analyze the behavior of FG structures.

Due to the above-mentioned favorable features, many studies have been conducted on the static and dynamic behavior of FG structures. The elasticity solutions of transversely and thermally loaded FG beams and also sandwich beams with FG cores were obtained by Sankar and his co-workers [4–6]. In their studies, the thermo mechanical properties of FGM were all assumed to vary exponentially through the thickness of the beam. Zhu and Sankar [7] solved the two-dimensional elasticity equations for the FG beam subjected to transverse loads using a mixed Fourier series – Galerkin method. Shi and Chen [8] studied the problem of a functionally graded piezoelectric cantilever beam subjected to different loadings. A pair of stress and induction functions was proposed in the form of polynomials and a set of analytical solutions for the beam subjected to different loadings was obtained. Nirmala et al. [9] derived analytical solutions for the thermo-elastic stresses in a three-layer composite beam with a middle FG layer. Using the meshless local Petrov–Galerkin method, Ching and Yen [10] presented numerical solutions for two-dimensional FG solids such as circular cylinders and simply supported beams subjected to either mechanical or thermal loads. Later, they used the meshless technique to obtain the transient thermo-elastic responses of FG beams under a non-uniform connective heat supply [11]. The free vibration of orthotropic FG beams with various end conditions was studied by Lu and Chen [12]. Wu et al. [13] obtained the closed-form natural frequencies of a simply supported FG beam with mass density and Young's modulus being polynomial functions of the axial coordinate. Aydogdu and Taskin [14] and Yang and Chen [15] analyzed the free vibration of FG beams. Kapuria et al. [16] presented a finite element model for static and free vibration responses of layered FG beams using a third-order theory and its experimental validation. A plane elasticity solution for a cantilever FG beam subjected to different static loads by means of the semi-inverse method was derived by Zhong and Yu [17]. Two-dimensional elasticity solutions for bending and free vibration of FG beams resting on Winkler–Pasternak elastic foundation were obtained by Ying et al. [18]. Li [19] proposed a new unified approach to investigate the static and free vibration behavior of Euler–Bernoulli and Timoshenko beams.

* Corresponding author. Address: Centre of Excellence for Research in Advanced Materials and Structures, Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Pardis St., Molasadra Ave., Vanak Sq., Tehran, Iran. Tel.: +98 2184063208; fax: +98 2188674748.
E-mail address: smrkhalili2005@gmail.com (S.M.R. Khalili).

0263-8223/$ - see front matter
doi:10.1016/j.compstruct.2010.02.012

Nomenclature

m	number of sampling time points per DQM time element
n_T	number of DQM time elements
A_{ij}	elements of first-order DQM weighting coefficient matrix
B_{ij}	elements of second-order DQM weighting coefficient matrix
E_t	modulus of elasticity (Young's modulus) at the top surface of the FG beam
E_b	modulus of elasticity (Young's modulus) at the bottom surface of the FG beam
E_{ratio}	ratio of Young's modulus at the top and bottom surfaces of the FG beam ($=E_t/E_b$)
ρ_t	density at the top surface of the beam
ρ_b	density at the bottom surface of the beam
ρ_{ratio}	ratio of density at the top and bottom surfaces of the FG beam ($=\rho_t/\rho_b$)
x	horizontal coordinate
z	vertical coordinate
t	time
k	power-law exponent
u	axial displacement of the beam
w	vertical displacement of the beam
$\phi(x)$, $\psi(x)$	Rayleigh–Ritz trial/test functions
M	mass of the moving load
v	velocity of the moving point load
g	gravitational acceleration
$[m^*]$, $[c^*]$, $[k^*]$	instantaneous mass, damping, and stiffness matrices due to the inertia of the moving load
$[M]$, $[C]$, $[K]$	structural mass, damping, and stiffness matrices

Based on the high-order sandwich panel theory, Rahmani et al. [20] developed a new model for free vibration analysis of sandwich beams with syntactic foam as a FG flexible core.

From the review of the literature, it is found that most of the researchers are interested in the free vibration analysis of FG structures, but few of them have paid attention to the forced vibration analysis of FG structures. On the other hand, to the authors' best knowledge, the information regarding the forced vibration responses of FG structures due to the moving loads is rare and this is the reason why this paper aims its study.

In the literature, one may refer to the work done by Yang et al. [21] or the recent study by Şimşek and Kocatürk [22]. In Ref. [21], the free and forced vibrations of cracked FG beams subjected to an axial force and a moving load were studied by using the modal technique. In the recent paper [22], the free and forced vibrations of a simply supported FG beam subjected to a concentrated moving harmonic load were analyzed. The transverse and axial displacements of the simply supported beam were first approximated by series of polynomial functions and by using the Lagrange equations the governing equations of motion of the FG beam were obtained in matrix form. The Newmark time integration scheme was then employed to solve the resulting dynamic equations. In both the papers [21,22], the effect of inertia of the moving load was ignored and the moving load was modeled as a simple moving force model (a force with constant [21] or harmonic magnitude [22] which moved along the FG beam). Besides, in Ref. [21] the material properties of the beam were only assumed to vary exponentially in the beam thickness direction and also the results of Ref. [22] were only given for simply supported FG beams.

The differential quadrature method (DQM), which was first introduced by Bellman et al. [23,24] in the early 1970s, is an alternative numerical solution technique for initial and/or boundary problems in engineering and mathematics. Its central idea is to approximate the derivative of a function at a discrete point with respect to a coordinate direction using a linear sum of all the function values at all discrete points along that direction [25,26]. Compared to the low-order methods, such as the finite element and finite difference methods, the DQ method can generate numerical results with high-order of accuracy by using a considerably smaller number of discrete points and therefore requiring relatively little computational effort. Another particular advantage of the DQ method lies in its ease of use and implementation.

The DQ method was initially proposed for the solution of initial-value problems [23], but little attention has been paid on this issue until recent years. Using the Hermite interpolation functions as the trial functions, Wu and Liu [27,28] proposed a generalized differential quadrature rule to solve linear and nonlinear initial-value problems. Shu and Yao [29] proposed a block Marching technique with DQ discretization for the solution of time-dependent problems. The time span was first divided into several blocks and the quadrature rule was then applied in each of the blocks. This technique reduces considerably the computational cost, since a smaller system of algebraic equations should be solved at each block (step). Tanaka and Chen [30,31] proposed a combined application of dual reciprocity boundary element method (DRBEM) and DQ method to transient diffusion and elastodynamic problems. In a series of papers on the solution of initial-value problems using the DQ method, Fung [32–34] introduced a modified DQ method to incorporate initial conditions. He also discussed in details, the stability property of the DQ method. It was emphasized that the accuracy and stability of the DQ method are dictated by the choice of sampling time points and also by the ways in which the initial conditions are incorporated into the solution process. He also showed that unconditionally stable algorithms using the DQ method can be obtained, if the initial conditions are incorporated by modifying the weighting coefficient matrices and if the number of sampling time points is small. Based on the construction of cubic cardinal spline functions, Zhong and Lan [35] developed a DQ time integration scheme and applied it to the solution of dynamic systems governed by Duffing-type nonlinear differential equations. Eftekhari et al. [36–40] proposed a coupled finite element – differential quadrature method for the solution of moving load class of problem. It has been claimed that the proposed mixed method is very efficient for the solution of time-dependent problems. Liu and Wang [41] presented an assessment of the DQ time integration scheme for nonlinear dynamic equations. It was shown that the DQ time integration scheme is reliable, computationally efficient and also suitable for time integrations over long time duration. But care should be taken in choosing a time step when applying the DQ method to nonlinear systems. In a series of papers, Civalek proposed a number of mixed methodologies for the nonlinear dynamic analysis of rectangular plates and multi-degree-of-freedom (MDOF) systems [42–44].

In this study a new mixed method, which combines the Rayleigh–Ritz method and the differential quadrature method (DQM), is proposed for forced vibration analysis of FG beams subjected to moving loads. The governing equations of motion of the FG beam are derived by using the Lagrange equations under the assumption of the Euler–Bernoulli beam theory. The Rayleigh–Ritz method is first employed to discretize the spatial partial derivatives, and the DQM is then applied to analogize the temporal derivatives. The resulting system of algebraic equations can be easily