



Nonlinear bending of FGM plates subjected to combined loading and resting on elastic foundations

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ABSTRACT

A nonlinear bending analysis is presented for a simply supported, functionally graded plate resting on an elastic foundation of Pasternak-type. The plate is exposed to elevated temperature and is subjected to a transverse uniform or sinusoidal load combined with initial compressive edge loads. Material properties are assumed to be temperature-dependent, and graded in the thickness direction according to a simple power-law distribution in terms of the volume fractions of the constituents. The formulations are based on a higher-order shear deformation plate theory and general von Kármán-type equation that includes the plate–foundation interaction and thermal effects. A two step perturbation technique is employed to determine the load–deflection and load–bending moment curves. The numerical illustrations concern nonlinear bending response of functional graded plates with two constituent materials resting on Pasternak elastic foundations from which results for Winkler elastic foundations are obtained as a limiting case. The results reveal that the characteristics of nonlinear bending are significantly influenced by foundation stiffness, temperature rise, transverse shear deformation, the character of in-plane boundary conditions and the amount of initial compressive load. In contrast, the effect of volume fraction index N becomes weaker when the plate is supported by an elastic foundation.

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1. Introduction

The demand for improved structural efficiency in aerospace structures has resulted a new class of materials, called functionally graded materials (FGMs). Typically, FGMs are made from a mixture of metals and ceramics and are further characterized by a smooth and continuous change of the mechanical properties from one surface to another. The ceramic constituents of FGMs are able to withstand high temperature environments due to their better thermal resistance characteristics, while the metal constituents provide stronger mechanical performance and reduce the possibility of catastrophic fracture. FGMs are now developed for an increased use as face sheets of sandwich structures, and in such a case the face sheet may be modeled as an FGM plate resting on an elastic foundation. Consequently, investigations on vibration characteristics of FGM plates resting on elastic foundations are identified as an interesting field of study in recent years [1–3].

Many studies for large deflection and/or nonlinear bending of FGM plates are available in the literature, see, for example, [4–11]. However, investigations in nonlinear bending analysis of FGM plates resting on an elastic foundation are limited in number.

Among those, Praveen and Reddy [4] analyzed nonlinear static and dynamic response of FGM plates subjected to transverse mechanical loads and a 1-D steady heat conduction by using finite element method. Shen [5,6] presented a nonlinear bending analysis of simply supported shear deformable FGM rectangular plates subjected to a transverse uniform and sinusoidal load in thermal environments or heat conduction. Yang and Shen [7] developed a semi-analytical-numerical method to examine the large deflection of shear deformable FGM rectangular plates subjected to combined mechanical and thermal loads under various boundary conditions. In their studies [5–7] heat conduction and/or temperature-dependent material properties were taken into account. Na and Kim [8] studied nonlinear bending of clamped FGM rectangular plates subjected to a transverse uniform pressure and thermal loads by using a 3-D finite element method. Ghannadpour and coworkers [9,10] studied large deflection of simply supported thin FGM plates subjected to a uniform transverse pressure based on the classical plate theory. Khabbazi et al. [11] studied nonlinear bending of simply supported FGM plates subjected to a uniform transverse pressure based on the first- and higher-order shear deformation plate theories. In their studies [9–11], however, the material properties were assumed to be independent of temperature. On the other hand, Yang and Shen [12] presented a large deflection analysis of thin FGM plates resting on a Pasternak elastic foundation subjected to combined transverse and in-plane loads, but the numerical results

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were only for a simple case of an FGM plate in a fixed thermal environment.

The present paper extends the previous works [5] to the case of FGM plates resting on a Pasternak elastic foundation. The plate is exposed to elevated temperature and is subjected to a transverse uniform or sinusoidal load combined with initial compressive edge loads. The material properties are assumed to be temperature-dependent, and graded in thickness direction according to a volume fraction power-law distribution. The formulations are based on a higher-order shear deformation plate theory and general von Kármán-type equation that includes the plate-foundation interaction and thermal effects. A two step perturbation technique is employed to determine the load–deflection and load–bending moment curves. The numerical illustrations show the nonlinear bending response of FGM plates resting on elastic foundations under different sets of loading conditions.

2. Theoretical development

Here we consider an FGM plate of length a , width b and thickness h , which rests on an elastic foundation. As usual, the reference coordinate system has its origin at the corner of the plate on the middle plane, as shown in Fig. 1. The plate is made from a mixture of ceramics and metals, the mixing ratio of which is varied continuously and smoothly in the Z direction. This is achieved by using a simple rule of mixture of composite materials. We assume that the composition is varied from the top to the bottom surface, i.e. the top surface ($Z = -h/2$) of the plate is ceramic-rich whereas the bottom surface ($Z = h/2$) is metal-rich. In such a way, the effective material properties P_f , like Young's modulus E or thermal expansion coefficient α , can be expressed as

$$P_f = P_c V_c + P_m V_m \quad (1)$$

in which P_c and P_m denote the temperature-dependent properties of the top and bottom surfaces of the plate, respectively, and V_c and V_m are the ceramic and metal volume fractions and are related by

$$V_c + V_m = 1 \quad (2)$$

and we assume the volume fraction V_m follows a simple power law as

$$V_m = \left(\frac{2Z + h}{2h} \right)^N \quad (3)$$

where the volume fraction index $N (0 \leq N \leq \infty)$ dictates the material variation profile through the plate thickness and may be varied to obtain the optimum distribution of component materials.

Since functionally graded structures are most commonly used in high temperature environment where significant changes in mechanical properties of the constituent materials are to be expected [13], it is essential to take into consideration this temperature-dependency for the accurate prediction of the mechanical responses. Thus, the effective Young's modulus E , and thermal expansion coefficient α are assumed to be functions of tempera-

ture, so that E and α are both temperature and position dependent. The Poisson's ratio ν_f depends weakly on temperature change and is assumed to be a constant. From Eqs. (1)–(3), one has

$$E = (E_m - E_c) \left(\frac{2Z + h}{2h} \right)^N + E_c, \quad \alpha = (\alpha_m - \alpha_c) \left(\frac{2Z + h}{2h} \right)^N + \alpha_c \quad (4)$$

It is evident that when $Z = -h/2$, $E = E_c$ and $\alpha = \alpha_c$, and when $Z = h/2$, $E = E_m$ and $\alpha = \alpha_m$.

The plate is exposed to elevated temperature and is subjected to a transverse uniform load $q = q_0$ or a sinusoidal load $q = q_0 \sin(\pi X/a) \sin(\pi Y/b)$ combined with initial compressive edge loads. Let $\bar{U}$, $\bar{V}$ and $\bar{W}$ be the plate displacements parallel to a right-hand set of axes (X, Y, Z) , where X is longitudinal and Z is perpendicular to the plate. $\bar{\Psi}_x$ and $\bar{\Psi}_y$ are the mid-plane rotations of the normals about the Y - and X -axes, respectively. As is commonly done, the foundation is assumed to be an attached foundation, meaning no part of the plate lifts off the foundation in the deformed region. The load–displacement relationship of the foundation is assumed to be $p = \bar{K}_1 \bar{W} - \bar{K}_2 \nabla^2 \bar{W}$, where p is the force per unit area, $\bar{K}_1$ is the Winkler foundation stiffness and $\bar{K}_2$ is a constant showing the effect of the shear interactions of the vertical elements, and ∇^2 is the Laplace operator in X and Y . Let $\bar{F}(X, Y)$ be the stress function for the stress resultants defined by $\bar{N}_x = \bar{F}_{,YY}$, $\bar{N}_y = \bar{F}_{,XX}$ and $\bar{N}_{xy} = -\bar{F}_{,XY}$, where a comma denotes partial differentiation with respect to the corresponding coordinates.

Reddy [14] developed a simple higher-order shear deformation plate theory. This theory assumes that the transverse shear strains are parabolically distributed across the plate thickness. The advantages of this theory over the first order shear deformation theory are that the number of independent unknowns ($\bar{U}$, $\bar{V}$, $\bar{W}$, $\bar{\Psi}_x$ and $\bar{\Psi}_y$) is the same as in the first order shear deformation theory, and no shear correction factors are required. Based on Reddy's higher-order shear deformation plate theory and including the plate-foundation interaction and thermal effects, the governing differential equations for an FGM plate undergoing moderately large rotations in the von Kármán sense can be derived as

$$\begin{aligned} & \tilde{L}_{11}(\bar{W}) - \tilde{L}_{12}(\bar{\Psi}_x) - \tilde{L}_{13}(\bar{\Psi}_y) + \tilde{L}_{14}(\bar{F}) - \tilde{L}_{15}(\bar{N}^T) \\ & - \tilde{L}_{16}(\bar{M}^T) + \bar{K}_1 \bar{W} - \bar{K}_2 \nabla^2 \bar{W} = \tilde{L}(\bar{W}, \bar{F}) + q \end{aligned} \quad (5)$$

$$\tilde{L}_{21}(\bar{F}) + \tilde{L}_{22}(\bar{\Psi}_x) + \tilde{L}_{23}(\bar{\Psi}_y) - \tilde{L}_{24}(\bar{W}) - \tilde{L}_{25}(\bar{N}^T) = -\frac{1}{2} \tilde{L}(\bar{W}, \bar{W}) \quad (6)$$

$$\tilde{L}_{31}(\bar{W}) + \tilde{L}_{32}(\bar{\Psi}_x) - \tilde{L}_{33}(\bar{\Psi}_y) + \tilde{L}_{34}(\bar{F}) - \tilde{L}_{35}(\bar{N}^T) - \tilde{L}_{36}(\bar{S}^T) = 0 \quad (7)$$

$$\tilde{L}_{41}(\bar{W}) - \tilde{L}_{42}(\bar{\Psi}_x) + \tilde{L}_{43}(\bar{\Psi}_y) + \tilde{L}_{44}(\bar{F}) - \tilde{L}_{45}(\bar{N}^T) - \tilde{L}_{46}(\bar{S}^T) = 0 \quad (8)$$

Note that the geometric nonlinearity in the von Kármán sense is given in terms of $\tilde{L}(\cdot)$ in Eqs. (5) and (6), and the other linear operators $\tilde{L}_{ij}(\cdot)$ are defined as in [5].

In the above equations, $\bar{N}^T$, $\bar{M}^T$, $\bar{S}^T$ and $\bar{P}^T$ are the forces, moments and higher order moments caused by the elevated temperature, and are defined by

$$\begin{bmatrix} \bar{N}_x^T & \bar{M}_x^T & \bar{P}_x^T \\ \bar{N}_y^T & \bar{M}_y^T & \bar{P}_y^T \\ \bar{N}_{xy}^T & \bar{M}_{xy}^T & \bar{P}_{xy}^T \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} A_x \\ A_y \\ A_{xy} \end{bmatrix} (1, Z, Z^3) \Delta T dZ \quad (9a)$$

and

$$\begin{bmatrix} \bar{S}_x^T \\ \bar{S}_y^T \\ \bar{S}_{xy}^T \end{bmatrix} = \begin{bmatrix} \bar{M}_x^T \\ \bar{M}_y^T \\ \bar{M}_{xy}^T \end{bmatrix} - \frac{4}{3h^2} \begin{bmatrix} \bar{P}_x^T \\ \bar{P}_y^T \\ \bar{P}_{xy}^T \end{bmatrix} \quad (9b)$$

where ΔT is temperature rise from some reference temperature at which there are no thermal strains, and

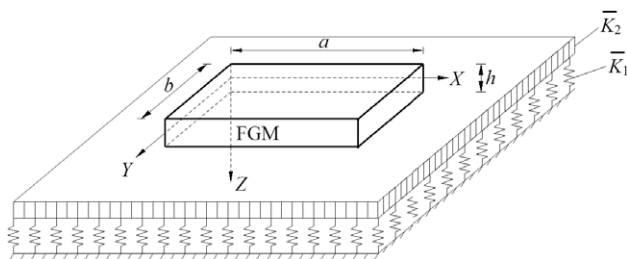


Fig. 1. An FGM rectangular plate resting on a Pasternak elastic foundation.

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