Available online at www.sciencedirect.com

ScienceDirect

Composite Structures 85 (2008) 311–325

COMPOSITE STRUCTURES

www.elsevier.com/locate/compstruct

An analytical–numerical approach to simulate the dynamic behaviour of arbitrarily laminated composite plates

Liz G. Nallim [a,*], Sergio Oller [b]

[a] CONICET, Facultad de Ingeniería, Universidad Nacional de Salta, Av. Bolivia 5150, 4400 Salta, Argentina
[b] Departamento de Resistencia de Materiales y Estructuras en la Ingeniería, Universidad Politécnica de Cataluña, Campus Norte UPC, Gran Capitán S/N, 08034 Barcelona, Spain

Available online 23 October 2007

Abstract

A general analytical–numerical approach developed for the dynamical analysis of unsymmetrically laminated plates of general quadrilateral planforms is presented in this work. An arbitrary quadrilateral thin flat laminate is mapped onto a square basic one, so that a unique macro-element is constructed for the whole plate. The Ritz method is applied to evaluate the governing equation in which the coupling effects of bending and stretching are contained. All possible transverse boundary conditions combining with the different in-plane constraints are considered in the analysis. The resulting algorithm possesses great flexibility, it is easy to program and it needs minimal input information. For these reasons, the proposed methodology results convenient for large scale structural design and analysis where repeated calculations are often required.

Keywords: Laminated plates; Elasticity; Free vibration

1. Introduction

Fibre-reinforced composite laminates are very important in many engineering applications. In addition to their high strength/light-weight, another important advantage of composite laminates is that structural properties can be tailored through changing the fibre angle and/or the number and sequence of plies. Particularly, laminated plates of different shapes made of advanced fibre-reinforced composite materials have many excellent advantages and are widely used as high-performance structural components. The accurate and efficient determination of the natural vibration frequencies and mode shapes of laminated plates components are essentials to the design and performance evaluation of a mechanical system. Moreover, the plate resonant frequencies and vibration mode shapes are often used to establish the dynamic response of complex engineering systems.

General angle-ply laminated plates exhibit various coupling effects, such as stretching–bending, stretching–shear and bending–twisting couplings, due to the anisotropy of the individual lamina and unsymmetrical layering [1]. Existence of these couplings and the general quadrilateral plate formulation are the source of analytical difficulties and of complicated mathematical structures for the boundary conditions making difficult the exact analysis for even the simplest cases. Consequently, most studies on these laminates employ approximate analytical or numerical methods.

A deep revision among the references about free vibration of thin unsymmetrically laminated composite plates reveals that most papers are concerned with rectangular plates (see e.g. [2–7]). Only limited literature can be found for unsymmetrically laminated plates of general quadrilateral shapes. In this topic, for instance, Liew and Lim [8] and Lim et al. [9] investigated the free vibration of trapezoidal multi-layered laminates with different combinations of boundary conditions using two-dimensional orthogonal

* Corresponding author. Tel.: +54 387 4258615; fax: +54 387 4255351.
E-mail addresses: lnallim@unsa.edu.ar (L.G. Nallim), sergio.oller@upc.edu (S. Oller).

0263-8223/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2007.10.031

polynomials as the trial functions in the Ritz method. For the study of free vibration of skew laminates Wang [10] used B-spline Rayleigh–Ritz method, while Krishna Reddy and Palaninathan [11] used a general high precision triangular plate bending finite element.

In general, it is observed that the Rayleigh–Ritz method has had a frequent application because of its high accuracy and relatively small computational cost, mainly related with the use of only one single super element in the whole process. The fundamental difficulty associated with the Ritz method in complex laminates is the choice of suitable functions to approximate the deflected shape. In a previous paper [12] the authors developed a general algorithm based on the Ritz method in conjunction with natural coordinates to express the geometry of laminated plates of general quadrilateral shapes in a simple form. That methodology was limited to the analysis of symmetrically laminated composite plates, where only the bending–twisting coupling needed to be included. In view of the aforementioned limitation, the present work has been undertaken to extend the mentioned algorithm to embrace laminates with unsymmetrical layering. The procedure involves the reformulation of the use of the geometric natural-to-Cartesian mapping concept in the Rayleigh–Ritz method including the effect of the bending–stretching coupling and the different transverse boundary conditions combining with four kinds of in-plane constraints. The general algorithm obtained offers much simplicity and automation in dealing with general laminated plates of several quadrilateral geometries and boundary conditions. It does not require any mesh discretization and it needs only very minimal input data for computations. Besides, the value of the analytical solution obtained here is that it allows getting insight into the behaviour on complex laminated plates.

To show the accuracy and the correctness of the presented method, convergence tests are carried out for several selected plate problems and, when it is possible, some results are compared with those published by other authors.

The algorithm developed can be applied to the analysis of a wide range of laminates with different shapes, aspect ratios, boundary conditions, number of layers, stacking sequences and angles of fibre orientation. The number of parameters is too high and the possibility of combination among them is infinite. For these reasons natural frequencies and nodal patterns of free vibration are presented for selected representative cases which can also be useful as benchmark comparison for future investigations in this topic.

2. Formulation

2.1. Energy functional components

Consider a flat, thin arbitrary-shaped quadrilateral fibre reinforced composite laminated plate of uniform thickness h as shown in Fig. 1a. The plane x–y is placed at the middle surface of the plate thickness, while z remains normal to it. In each layer of the laminate β denotes the angle of fibre orientation and the major and minor principal material axes are denoted by L and T, respectively. In consequence, the material constants are denoted by E_L, E_T, ν_{LT} and G_{LT}.

The fundamental displacements are the three mid-surface translational displacements u, v and w along the x, y and z directions, respectively. It is necessary that the two in-plane mid-surface translational displacements u and v are included in the analysis due to the coupling between in-plane and out-of-plane behaviour in laminates with unsymmetrical layup. Assuming that the Kirchhoff hypothesis holds, the translational displacements $\bar{u}, \bar{v}, \bar{w}$, at a general point in the laminate are given by:

$$\begin{aligned}
&\bar{u}(x,y,z,t) = u(x,y,t) - z\frac{\partial w(x,y,t)}{\partial x};\\
&\bar{v}(x,y,z,t) = v(x,y,t) - z\frac{\partial w(x,y,t)}{\partial y};\\
&\bar{w}(x,y,z,t) = w(x,y,t),
\end{aligned} \tag{1}$$

where t is the time dimension.

During free vibration, the displacements are assumed split in the spatial and temporal parts, being the last one periodic in time; i.e.,

$$\begin{aligned}
&u(x,y,t) = U(x,y)\sin\omega t,\\
&\quad v(x,y,t) = V(x,y)\sin\omega t, \quad w(x,y,t) = W(x,y)\sin\omega t,
\end{aligned} \tag{2}$$

where ω is the radian natural frequency.

Taking into account Eq. (2), the maximum strain energy of the unsymmetrically laminated plate can be written as

$$\begin{aligned}
U_{\max} = \frac{1}{2}\iint_R \Bigg[& A_{11}\left(\frac{\partial U}{\partial x}\right)^2 + 2A_{12}\frac{\partial U}{\partial x}\frac{\partial V}{\partial y} + A_{22}\left(\frac{\partial V}{\partial y}\right)^2\\
& + 2A_{16}\left(\frac{\partial U}{\partial x}\frac{\partial U}{\partial y} + \frac{\partial U}{\partial x}\frac{\partial V}{\partial x}\right)\\
& + 2A_{26}\left(\frac{\partial U}{\partial y}\frac{\partial V}{\partial y} + \frac{\partial V}{\partial y}\frac{\partial V}{\partial x}\right) + A_{66}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)^2\\
& - 2B_{11}\frac{\partial U}{\partial x}\frac{\partial^2 W}{\partial x^2} - 2B_{12}\left(\frac{\partial U}{\partial x}\frac{\partial^2 W}{\partial y^2} + \frac{\partial V}{\partial y}\frac{\partial^2 W}{\partial x^2}\right)\\
& - 2B_{22}\frac{\partial V}{\partial y}\frac{\partial^2 W}{\partial y^2}\\
& - 2B_{16}\left(\frac{\partial V}{\partial x}\frac{\partial^2 W}{\partial x^2} + \frac{\partial U}{\partial y}\frac{\partial^2 W}{\partial x^2} + 2\frac{\partial U}{\partial x}\frac{\partial^2 W}{\partial x\partial y}\right)\\
& - 2B_{26}\left(\frac{\partial U}{\partial y}\frac{\partial^2 W}{\partial y^2} + \frac{\partial V}{\partial x}\frac{\partial^2 W}{\partial y^2} + 2\frac{\partial V}{\partial y}\frac{\partial^2 W}{\partial x\partial y}\right)\\
& - 4B_{66}\frac{\partial^2 W}{\partial x\partial y}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right) + D_{11}\left(\frac{\partial^2 W}{\partial x^2}\right)^2\\
& + 2D_{12}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + D_{22}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4D_{16}\left(\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial x\partial y}\right)\\
& + 4D_{26}\left(\frac{\partial^2 W}{\partial y^2}\frac{\partial^2 W}{\partial x\partial y}\right) + 4D_{66}\left(\frac{\partial^2 W}{\partial x\partial y}\right)^2\Bigg]\mathrm{d}x\,\mathrm{d}y,
\end{aligned} \tag{3}$$