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A nonlinear zigzag theory for finite element analysis of highly shear-deformable laminated anisotropic shells

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Abstract

A hitherto unavailable highly accurate nonlinear finite element method (FEM)-based analysis technique for the prediction of large deformation response of highly shear-flexible arbitrarily laminated general shells of arbitrary geometry is presented. The nonlinear finite element utilizes the method of virtual work, and total Lagrangian (TL) formulation. It is based on the assumptions of transverse inextensibility, vanishing transverse normal strain, and layer-wise constant shear-angle theory (LCST), also known as the zigzag theory. The analysis includes fully nonlinear strain–displacement relations for the remaining five strain components. The components of displacements and stresses are computed using variable-node layer-elements of triangular plan-form.

Keywords: Zigzag theory; Transverse shear deformation; Nonlinear finite element; Triangular element; Curvilinear coordinates

1. Introduction

A reliable prediction of the response of thick laminated shell-type structures, made of high performance composite materials (e.g. graphite/epoxy, graphite/PEEK) necessitates the incorporation of the transverse shear deformation or cross-sectional warping into the formulation [1–3]. The majority of investigations pertaining to laminated shells are based on either classical lamination theory (CLT) or first-order shear deformation theory (FSDT), based on the Love–Kirchhoff and the Reissner–Mindlin hypotheses, respectively. The latter hypothesis implies constant cross-sectional warping [4,5], while the former means absence of the same. However, since the cross-sectional warping in a laminated shell may often be very severe (and non-uniform) due to the combined effects of thickness, lamination and the possibility of a high degree of anisotropy, use of the CLT or FSDT may lead to serious error. An improved theory known as the layer-wise constant shear-angle theory (LCST), also known as the zigzag theory, which allows for a piece (layer)-wise constant approximation of the non-uniform cross-sectional warping, leading to a quasi-3D elasticity solution in the limit, therefore, becomes an excellent practical alternative [1–3]. The primary advantage of the LCST-type theories lies in the fact that the ζ-dependence of the quantities of interest, such as displacements and stresses, can be separated from its surface-parallel (α, β) counterparts, and as a result, the through-layer-thickness integration of the stiffnesses can be performed in closed-form, which saves computation time (see Figs. 1,2) for coordinate system, shell geometry and the LCST-base laminated shell element of triangular plan-form.

It also has long been recognized that problems of post-buckling, collapse and compression fracture behaviors of shell-type structures cannot be predicted by linear theory as these phenomena are usually associated with large displacements and rotations. Nonlinear theory must be incorporated to compute such large displacements and rotations as obtained in such behaviors [6–14]. A numerical procedure, such as the finite element method (FEM), appears to be a viable practical approach because of the ease with which the problems of geometric, material and contact nonlinearities,

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Nomenclature

$[C^{(k)}]$	constitutive relation matrix at time t for the kth layer, relating the incremental second Piola–Kirchhoff stress and incremental Green–Lagrange strain
e_{ij}	covariant components of the Green–Lagrange strain tensor
g_{ij}, g^{ij}	covariant and contravariant components of the metric tensor of the undeformed body
g_α, g_β	first fundamental quantities of the undeformed shell reference surface
M	number of nodal points on a triangular element interface
N	number of layers
R_α, R_β	radii of curvature of the undeformed shell reference surface
$\overline{S}$	area of the bottom surface of a layer-element projected on the undeformed reference surface
t, t_k	total wall-thickness and kth layer-thickness respectively
$u_{(i)}$, $u^{(i)}$	physical components of the displacement vector
$\{\Delta\widehat{U}\}$	incremental nodal displacement vector
$\Delta\overline{W}^{(k)}$	incremental work of the kth layer per unit area of the undeformed reference surface
z	distance measured from the bottom (reference) surface of a laminated shell
α, β	lines of curvature coordinates attached to the undeformed reference surface of a laminated shell
$\varepsilon_{(ij)}$	physical components of the Green–Lagrange strain tensor
$\{\phi\}$	shape functions
θ^i	contravariant components of undeformed coordinates of a body
ξ_1, ξ_2, ξ_3	area coordinates of an element of triangular plan-form
ζ	distance measured from the bottom surface of a lamina
Left superscript	time t at which the quantities are being measured (time t is fictitious and used as an index for static problems, while real for a transient problem)
Left subscript	time, denoting the configuration, used as coordinate system

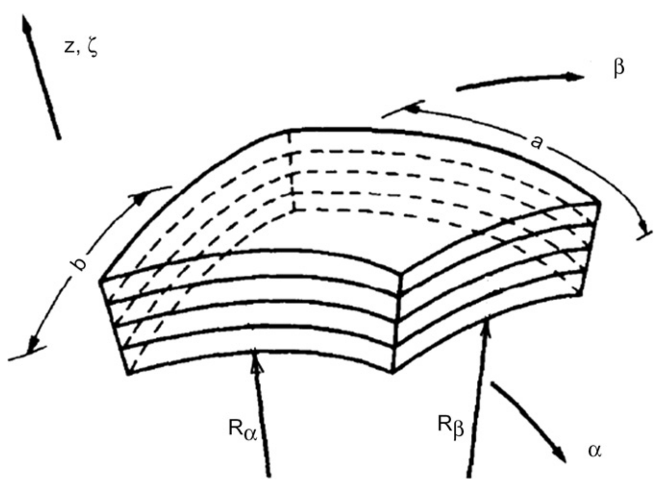

Fig. 1. A laminated thick shell and the associated coordinate system.

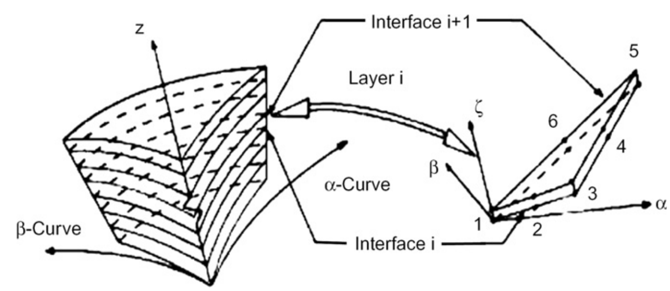

Fig. 2. Triangular composite shell element and mapping of its ith layer onto a straight-sided quadratic triangular element in the α–β plane ($M = 6$).

arbitrary shell geometry, non-uniform thickness, stiffeners and attachments, etc., can be handled by this method.

Noor and Hartley [15] were possibly the first to present large deformation analysis of laminated shells using mixed isoparametric triangular and quadrilateral elements. Their formulation is based on shallow shell theory that also includes the effect of interlaminar shear deformation. Librescu [16] introduced a higher-order shell theory into the formulation of geometric nonlinearity, by utilizing Hellinger–Reissner's variational principle. Chang and Sawamiphakdi [17] presented large deformation analysis of laminated shells using "degenerated" 3-D isoparametric elements, in which they utilized the updated Lagrangian (UL) nonlinear formulation. Epstein and Glockner [18] assumed a piecewise smooth displacement field to solve nonlinear finite element problems of multilayered shells. A 16-node isoparametric layer-element, with eight nodes on each of the top and bottom surfaces, was developed by Kim and Chaudhuri [19] for prediction of large deformation behavior of thick laminated composite cylindrical shells and panels. The analysis, based on the hypothesis of layer-wise linear displacement distribution through thickness (LLDT), accounts for fully nonlinear kinematic relations, in contrast to the commonly used von Karman nonlinear strain approximation, so that stable equilibrium paths in the advanced nonlinear regime can be accurately predicted. The special case of a moderately thick cylindrical shell isoparametric element, based on linear distribution