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Nonlinear dynamic response of piezoelectric elasto-plastic laminated plates with damage

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ABSTRACT

Based on the elasto-plastic mechanics and continuum damage theory, a yield criterion that is related to the spherical stress tensor is proposed to describe the mixed hardening of damaged orthotropic materials, and the dimensionless form of which is isomorphic with the Mises criterion for isotropic materials. Then, the incremental elasto-plastic damage constitutive equations and damage evolution equations of orthotropic materials are established. By using the classical nonlinear plate theory, the incremental nonlinear dynamic governing equations of the piezoelectric elasto-plastic laminated plates with damage are obtained, and the equations are solved by finite difference method and iteration method. In the numerical examples, the effect of damage evolution, piezoelectric effect and external loads on the nonlinear dynamic response of elasto-plastic laminated plates are discussed detailed. The numerical results show that the mechanical properties of the structures change remarkably when considering the damage, damage evolution and piezoelectric effects.

1. Introduction

Lighter weight, higher strength and higher energy dissipation capability makes the laminated composite materials extensively used in automotive and aerospace applications. As structures can still bear loads after exceed their yield limits, it is diseconomy to design structures by using the elastic theory, and the mechanical properties of structures in plastic stage are necessary to be analyzed. Moreover, due to the material defects or the technology problems in the machine-shaping, some damages may be created inevitably, inner or outside of the structures, these damages will decrease the stiffness and change the mechanical properties of the structures.

In recent years, the study of elasto-plastic constitutive relations of materials has achieved great progress. Zhu [1] developed an energy-based anisotropic damage model at finite strains for ductile fracture. Since the onset of damage and plastic yielding begins very early in the loading process, Taehyo and Voyiadjis [2] analyzed the damage and elasto-plastic behavior of metal matrix composites. Using the irreversible thermodynamics theory, Hayakawa [3] developed the constitutive and damage evolution equations for the elasto-plastic damage materials, and applied the equations to elucidate the damage process of tubular specimens of spheroidized graphite cast iron under tension and torsion. Voyiadjis and Deliktas [4] introduced the elasto-plastic damage model of rate-independent and rate-dependent, respectively, and based on the model, the stress–strain relationships of composite materials are calculated, then compared them to experimental results. By taking into account the kinematic description of damage, the anisotropic ductile models were proposed by Brünig [5], Brünig and Ricci [6]. Tham [7] presented an elasto-plastic damage model on the basis of irreversible thermodynamics and internal state variable formalism for the analysis of multi-layered composites. On the basis of Hill's yield criterion, Boutaous [8] developed an elasto-plastic damage model and analyzed the deformation of carbon fibre laminated plates. Within a thermodynamic framework and using continuum damage mechanics, Edlund and Volgers [9] derived a material model including the failure behavior for a thin unidirectional composite ply. Jason [10] thought the current elastic and elasto-plastic damage models failed to reproduce the unloading slopes during cyclic loading, so combined the plastic hardening model and isotropic damage model, a simple elasto-plastic damage model was established, and it was verified by three elementary tests. Based on the modified multiplicative decomposition of the left stretch tensor, the finite elastic-plastic deformations of hardening materials were analyzed by Ghavam and Naghdabadi [11]. But most of these models were built based on the Hill's yield criterion, which supposed the material yield is independent of the spherical stress tensor. Actually, under the acting of the spherical stress tensor, the structures will be distorted due to the different elastic constants in every principal direction. According to the Mises distortion energy yield criteria, the materials will get into plastic stage if the distortion energy reaches to a certain value, and the materials have

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apparent Bauchinger effect under the acting of the complex stresses. Hence, Hill's hypothesis is inconsistent with the practical situation. Against the flaw mentioned above, Yuan and Zheng [12] established a new yield criterion that relate to the spherical stress tensor, but the criterion was only suitable for undamaged materials.

Besides, the use of piezoelectric materials has received considerable attention in engineering due to the intrinsic direct and converse piezoelectric effects recently. Zhou et al. [13] investigated the dynamic control of laminated piezoelectric plates by finite element method. Based on Reddy's higher order shear deformation plate theory, Shen [14] investigated the postbuckling behaviors of laminated plates with piezoelectric actuators under complex loading conditions. Oh [15] studied the thermal postbuckling behavior of laminated plates with top and/or bottom actuators subjected to thermal and electrical loads. Qing [16] developed a new efficient numerical method and the dynamic analysis of composite laminates with piezoelectric layers. Ren [17,18] investigated the application of a piezoelectric actuator to control deformation of thin arbitrary lay-up composite laminates. Considered the effect of electromechanical coupling on the interlaminar stresses and the electric field strengths at free edges of laminated plates with piezoelectric material properties, Artel and Becker [19] compared the coupled and uncoupled piezoelectric analyses by using the finite element method. Shu [20] developed an accurate theory for laminated piezoelectric composite plates in cylindrical bending for free vibration analysis. Using the three-dimensional theory of elasticity, Kim and Lee [21] investigated the buckling of an orthotropic piezoelectric rectangular laminate with weak interfaces. Moreover, Li [22], Varelis and Saravanos [23], Zhou and Tzou [24], Li [25,26], Ray and Reddy [27], Wang et al. [28] also made great contributions to the field. But up to now, the existing research works are focused on the linear analysis of piezoelectric laminated structures in elastic deformation stage, and the analysis of nonlinear dynamic problems of elasto-plastic laminated structures with damage are not reported yet.

In present paper, under the assumptions that the plastic strains are compressible, the material yield is related to the spherical stress tensor, and the uniform dilatation produced by active stress will not influence the plastic deformation, a yield criterion that relevant to the spherical stress tensor for orthotropic damaged materials is presented. The dimensionless form of the yield criterion is isomorphic with the Mises criterion for isotropic materials. Furthermore, the incremental elasto-plastic damage constitutive equations and damage evolution equations for mixed hardening orthotropic materials are established. Then based on the classical nonlinear plate theory, the incremental nonlinear dynamic governing equations of piezoelectric elasto-plastic laminated plates with damage are derived. By adopting the finite difference method and iteration, the equations are solved. The numerical results show that the mechanical properties of the structures change remarkably when considering the damage, damage evolution and piezoelectric effects.

2. The elasto-plastic damage constitutive models of mixed hardening orthotropic materials

2.1. The elastic damage constitutive equations

Denote D_1, D_2, D_3 as the orthotropic principal damage variables of the materials along the direction of x, y, z, respectively, which can be defined as follows:

$$D_i = \frac{A_i - \widetilde{A}_i}{A_i} \quad (i = 1, 2, 3), \tag{1}$$

where A_1, A_2, A_3 are cross-sectional areas of undamaged materials along the direction of x, y, z, respectively, and $\widetilde{A}_i$ is the corresponding cross-sectional area after damage.

Based on the Lemaitre's equivalent strain principle, define σ and $\bar{\sigma}$ as the Cauchy stress and effective stress, respectively, the strain associated with the damaged material under the acting of applied stress σ is equivalent to the strain associated with its undamaged state under the acting of effective stress $\bar{\sigma}$. Using the tetradic denotes the orthotropic damage state, the effective stress can be expressed as

$$\bar{\sigma} = M(D) : \sigma \tag{2}$$

in which $M(D)$ is the effective damage tensor, it can be expressed as the six-order diagonal matrix [29], the diagonal elements are $M_{11}, M_{22}, M_{33}, M_{44}, M_{55}, M_{55}$ in turns, where $M_{11} = 1/(1-D_1)$, $M_{22} = 1/(1-D_2)$, $M_{33} = 1/(1-D_3)$, $M_{44} = 1/\sqrt{(1-D_2)(1-D_3)}$, $M_{55} = 1/\sqrt{(1-D_3)(1-D_1)}$, $M_{66} = 1/\sqrt{(1-D_1)(1-D_2)}$. When $D_1 = D_2 = D_3$, the orthotropic damage degenerates to isotropic damage, and when$D_1 = D_2$, the orthotropic damage degenerates to transverse isotropic damage.

According to the equivalent energy principle proposed by Sidoroff [30]: The complementary elastic energy associate with the damaged material under the acting of applied stress is in the same form of the complementary elastic energy associated with its undamaged state. Define C^e as the elastic tensor of undamaged material, and $\overline{C}^e$ as the effective elastic tensor of damaged material, the complementary elastic energy of the undamaged material is

$$W^e(\sigma, 0) = \frac{1}{2}\sigma^T : \overline{C}^{e^{-1}} : \sigma \tag{3}$$

and the complementary elastic energy of the damaged material is

$$W^e(\bar{\sigma}, D) = \frac{1}{2}\bar{\sigma}^T : (C^e)^{-1} : \bar{\sigma}. \tag{4}$$

Substituting Eq. (2) into Eq. (4), then compare it to Eq. (3), and define the effective elastic tensor as $\overline{C}^e = M^{-1} : C^e : (M^{-1})^T$, the elastic damage constitutive equation can be obtained as

$$d\sigma = \overline{C}^e : d\varepsilon^e. \tag{5}$$

2.2. The mixed hardening rule with damage

In the case of elasto-plastic deformation, we suppose: ① Spherical stress tensors produce plastic deformations, and the plastic strains are compressible. ② Uniform dilatation produced by active stresses will not influence the plastic deformation. ③ Yield surface moves and expands along with plastic deformations. ④ Dimensionless yield criterion of orthotropic material is isomorphic with the Mises criterion of isotropic material. Based on the above suppositions, the mixed hardening yield criterion of damaged materials can be written as

$$F_p = f(\bar{\sigma}_{ij},\ b_{ij}) - \kappa(\xi), \tag{6}$$

where $\bar{\sigma}_{ij}$ is the effective stress of damaged material. b_{ij} is the back stress, which denotes the transition of the center of the yield surface, and reflects the kinematic hardening. Hardening parameter $\kappa(\xi)$ denotes the size of yield surface, which is often set as equivalent stress. And ξ is an internal variable, often set as the effective plastic strain $\bar{\varepsilon}^p$.

For satisfying supposition ②, the strain components caused by the active stresses must meet $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33}$, that is

$$\frac{\bar{\sigma}_{11} - b_{11}}{\alpha_1} = \frac{\bar{\sigma}_{22} - b_{22}}{\alpha_2} = \frac{\bar{\sigma}_{33} - b_{33}}{\alpha_3}, \tag{7}$$

where $\alpha_1 = \overline{C}^e_{11} + \overline{C}^e_{12} + \overline{C}^e_{13}$, $\alpha_2 = \overline{C}^e_{21} + \overline{C}^e_{22} + \overline{C}^e_{23}$, $\alpha_3 = \overline{C}^e_{31} + \overline{C}^e_{32} + \overline{C}^e_{33}$. Suppose the principal directions of the material along the direction of x, y, z, the yield function $f(\bar{\sigma}_{ij} - b_{ij})$ can be given by