Composite Structures 88 (2009) 195–201

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Assessment of plate theories for initially stressed hybrid laminated plates

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ARTICLE INFO

Article history:
Available online 20 March 2008

Keywords:
Initial stress
Transverse shear strain
Normal strain
Plate theory

ABSTRACT

This study presents numerical solutions to hybrid laminate plates in a general state of non-uniform initial stress based on various plate theories. Firstly, the governing equations were formulated using Lo's high-order plate theory. The higher-order terms in Lo's displacement field were neglected to obtain various simple governing equations based on first-order theory and other higher-order theories. The various governing equations was used to investigate the natural frequencies and buckling loads of hybrid Al/GRFP/Al plates subjected to initial stresses. The effects of various parameters on the vibration frequency and buckling coefficient were examined. The results show that the transverse shear strain, normal strain, initial stress state and thickness ratio of Al to GRFP strongly affect the stability and vibration of hybrid laminate plates.

1. Introduction

Composite laminated materials are successfully used in aircraft industry and other engineering applications, mainly because of their high strength, high stiffness to weight ratio and excellent durability. The natural frequency and buckling load of a plate are important in the designs of structural components. Therefore, studies of vibration and buckling of plates have become very important in recent years. The accuracy of determinations of the vibration and buckling properties depends strongly on the displacement field of plate theory. Classical plate theory over-predicts buckling loads and natural frequencies [1,2]. Numerous plate theories have been proposed to improve on classical plate theory by considering the effects of the transverse strains [3–7]. In addition, numerous studies have addressed vibration and buckling based on various plate theories, to elucidate variations among the theories [8–11].

The cited studies examined only plates that were laminated with one material. The hybrid laminate plate consists of various materials. Xu et al. [12] presented the free vibration frequency of the hybrid composite plate. The plates consisted of fiber-reinforced cross-ply and piezothermoelastic layers. Numerical results showed the effects of the plate thickness and the location of the piezothermoelastic layers on the vibration frequencies. Barai and Durvasula [13] studied the vibration and buckling of curved plates using first-order plate theory. The effects of aspect ratio, stacking sequence and ply-orientation were studied. Benjeddou et al. [14] presented an analytical solution of simply supported piezoelectric adaptive plates. The proposed approach was numerically validated by a modal analysis of several hybrid plates with piezoceramic faces and an elastic cross-ply composite core.

Fiber-reinforced polyester laminates are widely used in engineering. However, when a composite laminate material is subjected to high temperature or corrosive moisture, its mechanical properties may be degraded. In such an environment, the material can be protected by covering the composite material with a layer of metallic material. Lee and Kim [15] presented the vibration of hybrid laminate plates based on classical plate theory. He investigated the effects of stacking sequence, aspect ratio, number of modes, number of layers and elastic properties on the vibration. Harras et al. [16] presented a theoretical model of the nonlinear free vibration of a hybrid composite plate. It consisted of alternating layers of metal and fiber-reinforced composites. The results indicated a great increase in the induced bending stresses at large deflections. Chen and Fung [17] presented the vibration behavior of a hybrid composite plate based on Mindlin plate theory. The results revealed that the plate vibration frequency ratios exceeded those obtained from the classical theory, because the latter neglects transverse shear and rotary inertia.

Residual stresses in the laminate plate are always produced by high curing temperature and pressure during manufacture. Besides those mentioned above, numerous studies on the vibration of initially stressed plates have published. Brunell and Robertson [18] studied the effects of an arbitrary initial stress on the vibrations of an isotropic plate using the average stress method and the variation method. Yamaki and Chiba [19] studied the vibration behavior of initially stressed laminated plates based on first-order plate theory. Nayak et al. [20] developed a shear deformable plate

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doi:10.1016/j.compstruct.2008.03.034

bending element based on a refined third order theory to analyse buckling and free vibration of initially stressed composite sandwich plates. Cheung et al. [21] used the finite strip method to analyze the nonlinear vibration of plates with initial stresses. Chen and Yang [22], Chen and Ku [23], Doong et al. [24,25] and Chen et al. [26,27] investigated the effects of initial stresses on the vibration of isotropic and composite plates. Based on various theories, Rogerson and Sandiford [28] presented an arbitrary strain energy function in the pre-stressed laminated plate at small amplitudes of vibration. However, studies of the vibration and stability of arbitrary initially stressed hybrid laminate plates based on various plate theories are few. This study extends a previous work to investigate the vibration and stability of laminated hybrid composite plates in an arbitrary state of initial stress. The governing equations of each plate model were derived using the average stress method based on Lo's displacement field [29]. Numerical results concerning natural frequency and buckling load were presented to investigate the differences among various plate theories. The effect of transverse shear and normal strains on the natural frequencies and buckling loads of an initially stressed plate based on various plate theories was discussed.

2. Displacement models

The geometry of the laminated plate was described in Cartesian coordinates x, y and z. The x–y plane was the middle plane and the z-axis was in the thickness direction. The displacements u, v and w at any position in the plate were represented as a Taylor series in terms of the thickness coordinate.

The laminate plate has been analyzed previously based on classical theory. Classical theory both neglects the effects of transverse shears and normal strain in the thickness direction and overestimates vibration frequencies and buckling loads. Numerous plate theories have been proposed to improve on classical plate theory by including the effects of the transverse shear strain in the plate. Reissner [30] and Mindlin and Deresiewicz [31] were the first to propose a plate theory that incorporates the effect of shear deformation. The displacement field of the Reissner–Mindlin theory was assumed to be of the form

$$\bar{u}_x(x,y,z,t) = u_x(x,y,t) + z\varphi_x(x,y,t), \quad (1)$$
$$\bar{u}_y(x,y,z,t) = u_y(x,y,t) + z\varphi_y(x,y,t), \quad (2)$$
$$\bar{u}_z(x,y,z,t) = w_z(x,y,t), \quad (3)$$

where $\bar{u}_x$, $\bar{u}_y$ and $\bar{u}_z$ were displacement functions in the x, y and z axes, respectively. First-order transverse shear deformation theory (FSDT5) includes five degrees of freedom u_x, v_y, w_z, φ_x and φ_y. The in-plane displacements u_x and u_y, the transverse displacement w_z and the angles of rotation of the normal to the middle plane φ_x, φ_y were time-dependent functions of the in-plane coordinates x and y. As the transverse shear strain is assumed to be a constant in Reissner–Mindlin plate theory, a correction factor is introduced into the shear stress resultants.

Essenburge [32] modified Eq. (3) into which the terms $z\varphi_z$ and $z^2\xi_z$ were added to take into account the influence of the transverse normal; his first-order transverse shear and higher-order transverse normal deformation theory is a seven-degree-of-freedom model denoted HSNT7 with u_x, u_y, w_z, φ_x, φ_y, ξ_x, ξ_y. The displacements were presented in the following form:

$$\bar{u}_x(x,y,z,t) = u_x(x,y,t) + z\varphi_x(x,y,t), \quad (4)$$
$$\bar{u}_y(x,y,z,t) = u_y(x,y,t) + z\varphi_y(x,y,t), \quad (5)$$
$$\bar{u}_z(x,y,z,t) = w_z(x,y,t) + z\varphi_z(x,y,t) + z^2\xi_z(x,y,t). \quad (6)$$

The model HSNT7 has the normal strain terms φ_z and ξ_z, which are not included in FSDT5. The shear correction factor in HSNT7 is the same as that in FSDT5.

The elasticity solution indicates that the transverse shear stresses varied with the square of the plate thickness; the in-plane displacements $\bar{u}_x$ and $\bar{u}_y$ were expanded to cubic functions of the thickness coordinate. Lo et al. [29] presented a theory that involved

$$\bar{u}_x(x,y,z,t) = u_x(x,y,t) + z\varphi_x(x,y,t) + z^2\xi_x(x,y,t) + z^3\phi_x(x,y,t), \quad (7)$$
$$\bar{u}_y(x,y,z,t) = u_y(x,y,t) + z\varphi_y(x,y,t) + z^2\xi_y(x,y,t) + z^3\phi_y(x,y,t), \quad (8)$$
$$\bar{u}_z(x,y,z,t) = w_z(x,y,t) + z\varphi_z(x,y,t) + z^2\xi_z(x,y,t). \quad (9)$$

Eqs. (10)–(12) were obtained from Eq. (9) by dropping the transverse normal strain terms that contained z and z^2. The model thus obtained was designated HSDT9, with nine degrees of freedom. Pandya and Kant [33] developed a higher-order transverse shear deformation theory from Lo's plate theory by dropping the transverse shear strain z and z^2 from Eq. (9)

$$\bar{u}_x(x,y,z,t) = u_x(x,y,t) + z\varphi_x(x,y,t) + z^2\xi_x(x,y,t) + z^3\phi_x(x,y,t), \quad (10)$$
$$\bar{u}_y(x,y,z,t) = u_y(x,y,t) + z\varphi_y(x,y,t) + z^2\xi_y(x,y,t) + z^3\phi_y(x,y,t), \quad (11)$$
$$\bar{u}_z(x,y,z,t) = w_z(x,y,t). \quad (12)$$

The four plate theories should be distinguished. FSDT5 includes constant shear strains and neglects normal strains. HSNT7 includes constant shear strains and variable normal strains. HSDT9 includes only parabolic shear strains and HSNT11 includes parabolic shear and variable normal strains. In this study, only HSNT11 was presented in detail to analyze the vibration and buckling of hybrid plates, as the other models involved the same procedure.

3. Theoretical formulation

Consider a body that is in static equilibrium, subjected to a time-varying incremental deformation, and in a state of non-uniform initial stress. A technique described by Bolotin [34] and Brunell and Robertson [35] was followed to obtain the governing equations using a perturbing technique:

$$(\sigma_{ij}\bar{u}_{s,j})_{,i} + [\bar{\sigma}_{ij}(\delta_{sj} + u_{s,j} + \bar{u}_{s,j})]_{,i} + \overline{F}_s + \Delta F_s = \rho\ddot{\bar{u}}_s, \quad (13)$$

where σ_{ij}, $\bar{\sigma}_{ij}$ and $\overline{F}_i$ are the initial stress, perturbing stress and body force, respectively. u_s and $\bar{u}_s$ represent the initial displacement and perturbing displacement, respectively.

The constitutive relations for each layer with respect to the laminate coordinate axes are

$$\begin{bmatrix} \bar{\sigma}_{xx} \\ \bar{\sigma}_{yy} \\ \bar{\sigma}_{zz} \\ \bar{\sigma}_{yz} \\ \bar{\sigma}_{zx} \\ \bar{\sigma}_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\varepsilon}_{zz} \\ \bar{\varepsilon}_{yz} \\ \bar{\varepsilon}_{zx} \\ \bar{\varepsilon}_{xy} \end{bmatrix}, \quad (14)$$

where C_{ij} are the components of the anisotropic stiffness matrix [36].

Eleven governing equations were obtained by substituting Lo's displacement field Eqs. (7)–(9) and the constitutive relation (14) into Eq. (13). The 11 governing equations are as follows:

$$Q_{1,x} + Q_{2,y} + (R_1 + R_5 + R_{17})_{,x} + (S_1 + S_6 + S_{17})_{,y} + f_x = I_1\ddot{u}_x + I_3\ddot{\xi}_x,$$
$$Q_{2,x} + Q_{3,y} + (R_9 + R_{13} + R_{21})_{,x} + (S_9 + S_{13} + S_{21})_{,y} + f_y = I_1\ddot{u}_y + I_3\ddot{\xi}_y,$$
$$Q_{4,x} + Q_{5,y} + (R_{30} + R_{33} + R_{25})_{,x} + (S_{30} + S_{33} + S_{25})_{,y} + f_z = I_1\ddot{w} + I_3\ddot{\xi}_z,$$