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Composite Structures 82 (2008) 609–621

COMPOSITE STRUCTURES

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Consistent higher-order free vibration analysis of composite sandwich plates

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Available online 28 February 2007

Abstract

The consistent higher-order dynamic formulation for foam-type (soft) core sandwich beams was extended to the case of composite sandwich plates. Eight dynamic governing equations and the corresponding boundary conditions were derived through the application of Hamilton's principle. The extended formulation was applied to the free vibration analysis of soft-core and honeycomb-core sandwich plates with anti-symmetric and symmetric lay-ups. The vibration results for the thin and thick composite sandwich plates obtained using the extended formulation were consistent with the predictions of the higher order mixed layerwise theory for laminated and sandwich plates. To simplify the formulation for the case of symmetric sandwich plates, the general dynamic formulation was decoupled into two systems of equations representing symmetric and anti-symmetric vibrations. The numerical study demonstrates the importance of the present formulation for the prediction of higher mode vibration response of composite sandwich plates.

Keywords: Vibration; Sandwich; Higher-order; Plates; Symmetric

1. Introduction

The free vibrations of composite sandwich plates have been extensively studied using classical analytical formulations [1–3], finite element analyses based on first- and higher-order shear deformation theories [4–7], and the spline finite point method (SFPM) [8]. In the majority of these studies, the sandwich core is assumed to be incompressible in the vertical direction. This assumption is practically accurate only for the vibration analysis of sandwich plates with a honeycomb core. However, in the case of a flexible sandwich core (for example, a foam core), this assumption will preclude modeling of the symmetric vibration modes where the two face sheets move out-of-phase. (For structural sandwich panels, the core is considered as vertically flexible when the ratio of Young's moduli of the face sheets to the core lies between 500 and 1000.) Furthermore, modeling of a flexible sandwich core with the aid of general-purpose commercial finite element software requires the use of 3-D solid elements, which consumes significant computational resources (memory and cpu time). The higher-order sandwich panel theory (HSAPT) [9] was derived to model the behavior of sandwich plates with a flexible core. This model is based on the nonlinear through-the-thickness displacement field in the core in both longitudinal and vertical directions. However, the corresponding acceleration field in the core is assumed to vary linearly with height, which introduces inconsistency in the formulation. For sandwich beams, this inconsistency has been overcome in the recently developed formulation [10] that accounts for a nonlinear acceleration field in the core.

Recently, a higher order mixed layerwise theory for laminated and sandwich plates was developed and described [11]. This theory accounts for nonlinear through-the-thickness distributions of the displacement field and continuity of the displacements and stresses at the interfaces between

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doi:10.1016/j.compstruct.2007.02.009

the adjacent layers. The vibration results of the mixed theory [11] were shown to be in good agreement with those obtained by the three-dimensional elasticity solutions of Srinivas and Rao [12].

In the present work, the one-dimensional beam formulation described in Ref. [10] is generalized to the two-dimensional plate analysis. The main difference between the present approach and the higher order mixed formulation in Ref. [11] is that in the latter case, the assumed through-the-thickness displacement field is used, whereas in the former, non-linear displacement field in the core is derived based on well-defined physical assumptions.

In what follows, a consistent free vibration formulation for composite sandwich plates is developed in terms of the system of governing partial differential equations and the corresponding boundary conditions. The derived formulation is applied to the vibration analysis of soft-core and honeycomb-core sandwich plates with anti-symmetric and symmetric lay-ups. Excellent agreement between the calculated vibration response for the thin and thick composite sandwich plates and the results of the higher order mixed layerwise theory in Ref. [11] is demonstrated. For the case of symmetric sandwich plates, simplification is achieved through the decoupling of the general formulation into two independent systems of equations representing symmetric and anti-symmetric motions. The numerical study of the free vibration response of composite sandwich plates based on the derived formulation follows next. Differences between the vibration responses of the soft-core sandwich plates with anti-symmetric and symmetric composite lay-ups are studied and discussed. The effect of honeycomb core modulus on the symmetric vibration response of sandwich plates with isotropic face sheets is investigated. Finally, the importance of the present formulation for the prediction of higher-frequency vibration response is discussed and conclusions are drawn.

2. Mathematical formulation

2.1. Assumptions

The present formulation is concerned with the linear vibration analysis of sandwich plates (see Fig. 1). The face sheets of a sandwich plate are assumed to behave as Kirchhoff thin plates with negligible shear deformations. The vertically compressible core layer is considered as an anti-plane, three-dimensional elastic medium with orthotropic out-of-plane shear properties [1]. Here, an antiplane assumption implies that the stresses in the core in planes parallel to the face sheets are neglected, and only the out-of-plane shear and normal stresses are accounted for in the analysis. This assumption is nearly exact for a honeycomb core, and is an appropriate approximation for an isotropic foam core, where the core material modulus is significantly lower than the modulus of the face sheets (see Section 1). Note that as a result of the compressibility of the core, the core height may change under loading and

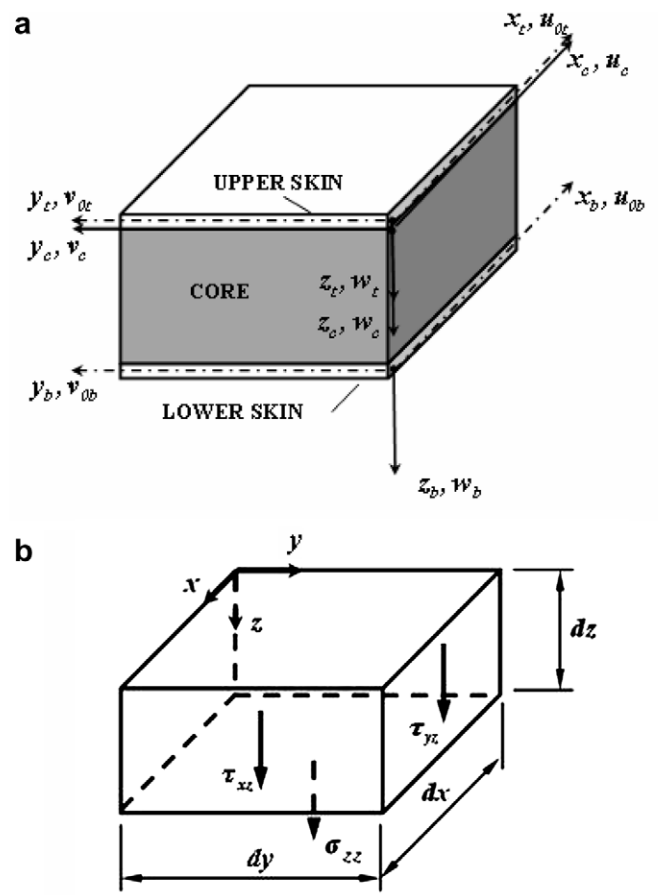

Fig. 1. Sandwich plate conventions: (a) geometry, coordinate systems and displacement functions of the sandwich plate and (b) stress field in the core.

the cross section may not remain planar. The interface layers between the face sheets and the core are assumed to provide perfect continuity of the deformations at the interfaces.

The acceleration fields of the face sheets are assumed to vary linearly with height, whereas the acceleration field of the compressible core varies nonlinearly with height in the plane and out-of-plane directions. Thus, the dynamic fields in the face sheets and core are consistent with the corresponding static patterns as normally assumed in dynamic analyses [13].

2.2. Equations of motion

The governing differential equations and the corresponding boundary conditions are derived here through the use of Hamilton's variational principle [14], namely,

$$\int_{t_1}^{t_2} (\delta T - \delta V)\mathrm{d}t = 0 \tag{1}$$

where T is the kinetic energy and V is the strain energy of a sandwich plate; t is the time coordinate; and t_1 and t_2 are the values of the time coordinate at the beginning and end of the motion, respectively.

The strain energy of the composite face sheets is given by [7]