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Stress analysis and material tailoring in isotropic linear thermoelastic incompressible functionally graded rotating disks of variable thickness

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ABSTRACT

We analyze axisymmetric deformations of a rotating disk with its thickness, mass density, thermal expansion coefficient and shear modulus varying in the radial direction. The disk is made of a rubberlike material that is modeled as isotropic, linear thermoelastic and incompressible. We note that the hydrostatic pressure in the constitutive relation of the material is to be determined as a part of the solution of the problem since it cannot be determined from the strain field. The problem is analyzed by using an Airy stress function φ. The non-homogeneous ordinary differential equation with variable coefficients for φ is solved either analytically or numerically by the differential quadrature method. We have also analyzed the challenging problem of tailoring the variation of either the shear modulus or the thermal expansion coefficient in the radial direction so that a linear combination of the hoop stress and the radial stress is constant in the disk. For a rotating annular disk we present the explicit expression of the thermal expansion coefficient for the hoop stress to be uniform within the disk. For a rotating solid disk we give the exact expressions for the shear modulus and the thermal expansion coefficient as functions of the radial coordinate so as to achieve constant hoop stress. Numerical results for a few typical problems are presented to illuminate effects of material inhomogeneities on deformations of a hollow and a solid rotating disk.

1. Introduction

Functionally graded materials (FGMs) are composites in which the volume fraction, sizes, and shapes of material constituents can be varied to get desired smooth spatial variations of macroscopic properties such as the elastic modulus, the mass density, the heat conductivity, etc. to optimize their performance. The FGMs abound in nature, e.g., human teeth, bamboo stick, sea shell. Engineered FGMs include ceramic–metal and fiber-reinforced polymeric composites, concrete, and rubberlike materials [1–5]. Vulcanized rubber components typically exhibit a spatial variation of mechanical properties caused either by thermal gradients during their fabrication or chemical changes induced due to interaction with the environment during their service [6–8]. For example, in a commercial butyl rubber sheet and a chlorosulfonated polyethylene cable jacketing material the shear modulus was found to be a quadratic function of the radius [9]. Note that during the fabrication (vulcanization) of thick rubber parts the central portion is cured less than the material near the boundary surfaces unless the vulcanization time is sufficiently large [8,9]. Thus the shear modulus at the center is less than that at the surfaces.

Rubberlike materials are widely used in aerospace, automotive, and biomedical fields. They are usually regarded as incompressible, can thus undergo only isochoric or volume preserving deformations, and their constitutive relation involves hydrostatic pressure that cannot be determined from the deformation field but is to be found as a part of the solution of the boundary-value problem (BVP). The BVPs for functionally graded incompressible materials (FGIMs) are challenging since the governing differential equations have variable coefficients and it is difficult to find their exact solutions. In general, the solution of a BVP for a structure composed of an FGIM cannot be obtained from that of the corresponding problem for a compressible material by setting Poisson's ratio equal to 0.5. Furthermore, the solution for a plane stress problem cannot be obtained from that for a plane strain problem by modifying Young's modulus E and Poisson's ratio ν. We briefly review below the literature on FG rotating disks and other works for FGIMs.

Deformations of a rotating disk composed of a linear elastic, isotropic and homogeneous material have been studied thoroughly [10] and those of a FG rotating disk have been investigated by Horgan and Chan [11] by assuming that E is a power-law function of the radius r. Jahed et al. [12] presented a procedure for minimum

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mass design of rotating disks with variable material properties and operating at a high temperature. Eraslan and Akis [13] obtained closed-form solutions for FG rotating solid shafts and disks by assuming that E is either an exponential or a parabolic function of r. Kordkheili and Naghdabadi [14] used a semi-analytical approach to analyze axisymmetric thermoelastic deformations of hollow and solid rotating FG disks with the thermomechanical properties given by a power-law function of r. You et al. [15] derived a closed-form solution for FG rotating disks subjected to a uniform temperature change by taking E, the thermal expansion coefficient and the mass density to vary according to power-law functions of r. Hojjati and Jafari [16] introduced two analytical methods, namely homotopy perturbation and Adomian's decomposition, to find stresses and displacements in rotating annular elastic disks with uniform and variable thicknesses and mass densities. Bayat et al. [17–20] solved the elastic and thermoelastic problems for FG rotating disks with the assumption that the material properties and disk thickness are given by power-law functions of r and the temperature field is steady. Vullo and Vivio [21,22] studied stresses and strains in variable thickness annular and solid rotating elastic disks subjected to thermal loads and having a variable density along the radius. Zenkour [23,24] investigated the stress distribution in rotating three-layer sandwich solid disks with face sheets made of different isotropic materials and a FG core.

For FGIMs, Batra [25] numerically studied axisymmetric static deformations of a Mooney–Rivlin cylinder with material parameters taken to be quadratic functions of r. Batra et al. [26–28] have found that the hoop stress in a cylinder is constant if the shear modulus is a linear function of the radius. Bilgili et al. [4–6,29,30] have analyzed shearing deformations of an inhomogeneous rubberlike slab or tube subjected to a thermal gradient across its thickness or radius.

We note that most problems for FGMs have been studied by assuming that material parameters vary either as a power-law function or an exponential function in one direction. Here we study infinitesimal deformations of a rotating disk composed of an isotropic linear thermoelastic FGIM when the shear modulus is an arbitrary smooth function of r. For a few specific variations of the shear modulus we provide exact solutions, and for a general smooth variation we solve the problem numerically by the differential quadrature method (DQM) [31]. Furthermore, we study the material tailoring problem and find either the shear modulus or the thermal expansion coefficient as a function of r to achieve a desired radial variation of stresses. For plane strain axisymmetric deformations of an FG cylinder composed of an orthotropic compressible material, Leissa and Vagins [32] assumed that all material moduli are proportional to each other and found their spatial variation so to make either the hoop stress or the shear stress uniform in the cylinder.

Qian and Batra [33] used a higher-order shear and normal deformable plate theory [34] to find the spatial variation along the axial and the thickness directions of the two constituents in a FG cantilever plate to optimize the fundamental frequency. The through-the-thickness variation of the fiber orientation angle in a fiber-reinforced laminated composite plate to optimize one of the first five lowest frequencies of a rectangular plate under different boundary conditions is given in [35], and the axial variation of the shear modulus to control the angle of twist per unit length for the torsion of a FG cylinder in [36]. Batra [37] has derived a higher-order plate theory for FGIM plates, and an exact solution for frequencies of a simply supported plate made of an incompressible material is provided in [38].

The rest of the paper is organized as follows. Section 2 gives the problem formulation, and Section 3 presents exact solutions for stresses and the radial displacement in rotating disks with the shear modulus given by either a power-law or an exponential function of r. For a general variation of the shear modulus, the solution of the problem by the DQM is provided. The material tailoring problem is studied in Section 4, and Section 5 gives numerical examples both for the material tailoring problem, and the stress and the displacement variations in FGIM rotating disks of variable thickness under different boundary conditions. Section 6 summarizes conclusions of the work.

2. Problem formulation

Consider a circular disk of thickness $h(r) > 0$ varying in the radial direction only, having inner radius, r_{in}, and outer radius, r_{ou}, and rotating at a constant angular velocity, ω, about the centroidal axis perpendicular to the plane of the disk, as shown in Fig. 1. Assuming that $r_{ou}/h_{\max} \geqslant 10$ we regard the state of deformation in it to be that of plane stress, and investigate the effect of ω on stresses induced by taking its deformations to be axisymmetric; here h_{max} equals the maximum thickness of the disk. We use cylindrical coordinate system (r, θ, z) with the origin at the disk center and the z-axis perpendicular to the plane of the disk.

In the absence of gravitational forces, the equation of equilibrium in the radial direction is [10]

$$\frac{d}{dr}(h(r)r\sigma_{rr}) - h(r)\sigma_{\theta\theta} + h(r)\rho(r)\omega^2 r^2 = 0, \tag{1}$$

where σ_{rr} and $\sigma_{\theta\theta}$ are, respectively, the radial and the hoop stresses at a point, and $\rho(r)$ is the mass density. We solve the problem for the following three sets of boundary conditions on the inner and the outer surfaces of the disk.

Case 1: hollow disk with the inner and the outer surfaces traction free:

$$\sigma_{rr}(r_{in}) = 0.0, \quad \sigma_{rr}(r_{ou}) = 0.0. \tag{2a, b}$$

Case 2: hollow disk with the inner surface fixed and the outer surface traction free:

$$u_r(r_{in}) = 0.0, \quad \sigma_{rr}(r_{ou}) = 0.0. \tag{2c, d}$$

Case 3: solid disk with the outer surface traction free:

$$u_r(0.0) = 0.0, \quad \sigma_{rr}(r_{ou}) = 0.0, \tag{2e, f}$$

where u_r is the radial displacement of a point.

Assuming that deformations are infinitesimal, the in-plane radial and hoop strains, ε_{rr} and $\varepsilon_{\theta\theta}$, respectively, are related to u_r by

$$\varepsilon_{rr} = \frac{du_r}{dr}, \quad \varepsilon_{\theta\theta} = \frac{u_r}{r}. \tag{3a, b}$$

The axial strain ε_{zz} in the z-direction is generally non-zero. The compatibility equation in terms of strains is

$$\frac{d}{dr}(r\varepsilon_{\theta\theta}) - \varepsilon_{rr} = 0. \tag{4}$$

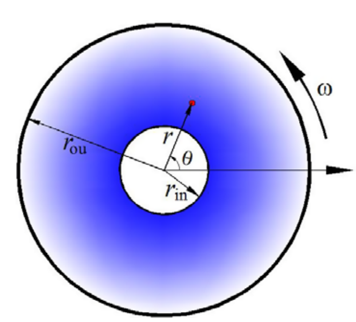

Fig. 1. Schematic sketch of the problem studied.