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Bending and free vibration response of layered functionally graded beams: A theoretical model and its experimental validation

S. Kapuria [a,*], M. Bhattacharyya [a], A.N. Kumar [b]

[a] Applied Mechanics Department, I.I.T. Delhi, Hauz Khas, New Delhi, India
[b] University of Ottawa/CISTECH Corporation, Ottawa, ON, Canada

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Abstract

A third order zigzag theory based model for layered functionally graded beams in conjunction with the modified rule of mixtures (MROM) for effective modulus of elasticity is validated through experiments for static and free vibration response. Two systems, Al/SiC and Ni/Al_2O_3, fabricated using powder metallurgy and thermal spraying techniques respectively, are considered for the validation. The theoretical predictions for the layered beams with the ceramic content varying from 0% to 40% are compared with the experimental data for the static deflection under simply-supported and cantilever boundary conditions, and for the natural frequencies under cantilever and clamped-clamped boundary conditions. The predictions using the MROM are found to be in close agreement with the experiments for both systems, whereas the linear rule of mixtures based property estimates lead to highly erroneous results. The effect of number of layers on the accuracy of the theoretical model is discussed. The accuracy of the predicted results gives confidence on the values of stress to strain transfer ratio used in the MROM for the two systems in the layered fabrication context, and also demonstrates the capability of the zigzag theory in accurately modelling the mechanics of such beams.

Keywords: Functionally graded material; Modified rule of mixtures; Free vibration; Experiment; Finite element; Zigzag theory

1. Introduction

The concept of functionally graded material (FGM) emerged from the need to fabricate a new composite for high temperature structural applications by using a heat resistant ceramic on the high temperature side and a metal on the low temperature side to provide mechanical toughness [1]. The gradient compositional variation of the constituents from one surface to the other provides an elegant solution to the problem of high transverse shear stresses that are induced when two dissimilar materials with large difference in material properties are bonded. Since its inception, the concept has found many potential applications like thermal and corrosion barriers, dental and orthopedic implants, plasma facing bio materials, sensors, lightweight armour material with high ballistic efficiency etc. [2–5]. The gradation of the volume fractions of the constituents along the thickness direction can have a continuous or a step-wise variation [6].

Several studies have been performed to analyze the static and dynamic behavior of functionally graded beams, plates and shells. The various models available for the analysis of FGM structures can be categorized based on two criteria (i) the type of displacement field approximations across the thickness (kinematic modelling) and (ii) the direct use of assumed variations of the material properties across the thickness or use of a material model for computing the effective properties at a point with given volume fractions of the constituents. The accuracy of prediction of response of FGM structures will depend on both the kinematic modelling and the correctness of estimated effective material properties of the two-phase system. Exact analytical solutions based on three

* Corresponding author. Fax.: +91 11 26581119.
E-mail address: kapuria@am.iitd.ac.in (S. Kapuria).

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dimensional (3D) elasticity have been presented for simply supported functionally graded infinite panel [7] and rectangular plate [8] under mechanical load, and for flat panel under transient thermal load [9], considering an assumed exponential variation of Young's modulus and thermal expansion coefficient, and constant Poisson's ratio over the thickness. Vel and Batra [10] have presented a 3D exact solution for free and forced vibrations of simply supported rectangular functionally graded plates, considering the Mori–Tanaka [11] and the self consistent [12] methods for computing the effective Young's modulus and Poisson's ratio with power law variation of the volume fractions of the constituents across the thickness. Such 3D analytical solutions are available only for specific geometry and boundary conditions. For analysis of structures or arbitrary shape and boundary conditions, several 2D models for plates and shells and 1D models for beams have been developed. The classical plate theory (CPT) has been employed for static bending [13] and transient [14] nonlinear response of FGM plates, using a power law variation for the volume fractions and employing linear rule of mixtures [15] for computing all effective material properties. The classical theory neglects shear deformation. Reddy and his coworkers have employed first order shear deformation theory (FSDT) (the simplest one to incorporate shear deformation effect) for thermoelastic finite element analysis of FGM plates [16] and for axisymmetric bending of circular FGM plates [17] using the linear rule of mixtures (ROM) for computing effective material properties with a power law variation of the volume fractions. Reddy [18], and Yang and Shen [19] have employed the refined third order theory (TOT) with an assumed power law variation for Young's modulus, density and thermal expansion coefficient and a constant Poisson's ratio for static and dynamic response of FGM plates under thermal loading. The same material model has been used by Chakraborty et al. [20], who presented a FSDT based shear locking free element for FGM beams. To the best of the authors' knowledge, no experimental validation of the theoretical predictions of static and dynamic response of FGM beams and plates has been reported in open literature.

In a recent study, the authors [21] showed that the experimentally obtained values for the modulus of elasticity for Al/SiC composite system with different volume fractions of the constituents are considerably lower than those predicted using the ROM. It was also shown that the modified rule of mixtures (MROM) proposed by Tomota et al. [22] for two-phase systems, which involves a parameter for the stress to strain transfer between the two phases, can be used to accurately predict the Young's modulus of this system. This parameter was found to be consistent across different volume fractions of Al and SiC.

This paper presents a finite element model for the dynamic analysis of layered FGM beams using an efficient third order zigzag theory [23] for the layerwise mechanics and the MROM for estimating the effective modulus of elasticity, and its experimental validation for the static deflection and natural frequencies of two different FGM systems under various boundary conditions. The two FGM systems considered are Al/SiC and Ni/Al_2O_3, both of which have generated considerable interest as heat resistent materials [24,25]. Powder processes are ideally suited for fabricating gradient materials because of the excellent microstructural control and the versatility in these techniques. Several studies have been reported on the fabrication of ceramic–metal gradient materials by different powder metallurgy methods such as consolidation of powder stack [26,27], consolidation of green sheet lamination [28,29], powder slurry spraying [30], and combustion synthesis [31]. Reviews of these methods have been recently presented by Kieback et al. [6] and Watanabe et al. [32]. In the present study, Al/SiC FGM beam samples with three and five layers were fabricated using the powder metallurgy route following consolidation of powder stack with cold isostatic pressing (CIP) followed by sintering. The study on the beams with two different numbers of layers illustrates the effect of the gradation step size on the applicability of the theoretical model. Five-layered Ni/Al_2O_3 beam samples were prepared using combustion powder thermal spray process [33]. The two systems fabricated with different methods ascertain the consistency of the theoretical model.

2. Third order zigzag beam theory

The zigzag one dimensional (1D) beam theory presented by Kapuria et al. [23] is used to predict the response of the layered FGM beam. Consistent with the fabrication process, the FGM is modelled in this theory as a laminate of multiple perfectly bonded layers of isotropic material of layerwise constant composition. Let the beam be made of L such layers of discretely varying compositions. In general, the volume fractions V_c and V_m of the ceramic and the metal are assumed to vary along the thickness direction z according to a power law:

$$V_m(z) = \left\{ \begin{array}{ll} 1-(z/h+0.5)^M & \text{for } M \geqslant 1 \\ \{1-(z/h+0.5)\}^{1/M} & \text{for } M \leqslant 1 \end{array} \right\},$$
$$V_c(z) = 1 - V_m(z), \quad (1)$$

where M is non-negative real number called the inhomogeneity parameter. $M = 1$ gives a linear variation of V_c and V_m. The volume fraction V_m of a given layer is computed as the area average of V_m–z/h curve for the layer thickness.

Let the length along direction x, width along direction y and thickness in direction z of the FGM beam be a, b and h, respectively. The z-coordinate of the bottom surface of the kth layer from bottom is denoted as z_{k-1}. For a beam with a small width, the following usual assumptions are made: assume state of plane stress ($\sigma_y = \tau_{yz} = \tau_{xy} = 0$), neglect the transverse normal stress ($\sigma_z \simeq 0$) and assume