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# Exact solution for thermo-elastic response of functionally graded rectangular plates

A. Alibeigloo *

*Meh. Eng. Dep., Bu-Ali Sina University, P.O. Box: 65175-4161 Hamedan, Iran*

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ABSTRACT

Three-dimensional thermo-elastic analysis of functionally graded (FG) rectangular plates with simply supported edges subjected to thermo-mechanical loads are carried out in this paper. The thermo-elastic constants of the plate were assumed to vary exponentially through the thickness, and the Poisson ratio was held constant. Analytical solutions for the temperature, stress and displacement fields are derived by using the Fourier series and state-space method. To verify the accuracy of the present work, a comparison is made with previously published results. The effects of temperature change, applied mechanical load, gradient index, aspect ratio and thickness to length ratio on the behavior of the plate are examined.

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## 1. Introduction

Functionally graded materials (FGMs) are heterogeneous composite materials with gradient compositional variation of the constituents from one surface of the material to the other which results in continuously varying material properties. FGMs were initially designed as thermal barrier materials for aerospace structures and fusion reactors. They are now developed for the general use as structural components in high-temperature environments, and consequently many studies on thermo-mechanical characteristics of FGM plates are available in the literature. Wang and Tarn [1,2] developed three-dimensional analysis of inhomogeneous plate by using an asymptotic expansion method. Instead of exactly solving the heat conduction equation, they presumed a temperature field a priori. for FGMs, Aboudi et al. [3] analyzed the thermo-elastic response of FGM plate by using the Higher Order Theory. Transient nonlinear thermo-elastic behavior of a FG ceramic/metal plate was investigated by Praveen and Reddy [4] by applying the von Karman plate theory and the finite element method. Reddy and Chin [5] carried out theoretical as well as finite element analyses of the thermo-mechanical behavior of FGM cylinders and plates. Ootao and Tanigawa [6] obtained analytical solutions for unsteady-state thermal stress of FG rectangular plate subjected to partial heating. Thermo-mechanical deformations of a FG elliptic plate with rigidly clamped edges was analyzed by Cheng and Batra [7]. They found that through thickness distributions of the in-plane displacements and transverse shear stresses in a FG plate do not agree with those assumed in classical and shear deformation plate theories. The thermo-elastic behavior of an orthotropic inhomogeneous rectangular plate was carried out by Kawamura et al. [8]. Reddy and Cheng [9] gave three-dimensional analytical solution for thermo-mechanical response of simply supported, FG, rectangular plate by using an asymptotic expansion method. Applying Mori–Tanaka's method and assuming a power law volume fraction distribution of the constituents, they investigated the influence of the exponent of the volume fraction law on the structural response under pure thermal or mechanical loads. Three-dimensional deformations of a simply supported FG rectangular plate subjected to mechanical and thermal loads on its top and/or bottom surfaces have been analyzed by Vel and Batra [10]. Vel and Batra [11] obtained three-dimensional transient thermal stresses of FG rectangular plate by extending the analytical technique reported in [10]. Tsukamoto [12] examined thermal stresses in a ceramic–metal plate subjected to through-thickness heat flow using the Mori–Tanaka scheme and the classical laminated plate theory. Batra and Qian [13] presented transient response of a thick FG plate, by using higher-order shear and normal deformable plate theory and a meshless local Petrov–Galerkin method. The exact solution for transient temperature and thermal stresses of a FG strip with simply supported edges due to a non-uniform heat supply in the width direction under the plane strain condition was obtained by Ootao and Tanigawa [14]. Ferreira et al. [15] derived static response of a thick plate by using a meshless local Pettrov–Galerkin method based on third-order shear deformation theory. Thermo-elastic solution for transient thermal stresses of FG rectangular plate due to non-uniform heat supply was presented by Ootao and Tanigawa [16] using series expansions of the Bessel functions. Three-dimensional exact solution for the transient thermo-elastic response of an orthotropic FG rectangular plate with simply supported edges due to a non-uniform heat supply was obtained by Ootao and Tanigawa [17]. Thermo-elastic deformations of a simply supported, FG, rectangular plate was derived by Brischetto et al. [18] by using principle of

* Tel.: +98 811 8283022; fax: +98 811 8257400.
*E-mail address:* beigloo@basu.ac.ir

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**Nomenclature**

| | |
|---|---|
| $a, b$ | plate dimension in $x$ and $y$ directions |
| $E, \alpha, \lambda$ | Young's modulus, thermal expansion coefficient and thermal conductivity coefficient, respectively |
| $E_0, \alpha_0, \lambda_0$ | Young's modulus, thermal expansion coefficient and thermal conductivity coefficient at the bottom surface, respectively |
| $E_h, \alpha_h, \lambda_h$ | Young's modulus, thermal expansion coefficient and thermal conductivity coefficient at the top surface, respectively |
| $h$ | plate thickness |
| $n, m$ | number of half waves in $x$ and $y$ direction |
| $U, V, W$ | displacement in $x$-, $y$- and $z$-directions, respectively |
| $\gamma_{zy}, \gamma_{zx}, \gamma_{xy}$ | shear strains |
| $\sigma_i$ ($i = x, y, z$) | normal stresses |
| $\varepsilon_i$ ($i = x, y, z$) | normal strains |
| $\tau_{zy}, \tau_{zx}, \tau_{xy}$ | shear stresses |
| $\lambda, \mu$ | Lame coefficients |
| $\delta$ | state variables |

virtual displacements. Matsunaga [19] presented thermo-elasticity solution of FG rectangular plate with simply supported edges and subjected to thermal and mechanical loads by using a two-dimensional higher-order shear deformation theory. As the aforementioned works show, the exact solution for FGM rectangular plate subjected to thermo-mechanical load by making the use of state-space method has not been yet considered and the present work attempts to do this.

In this paper, thermo-elastic solution for FG plate with finite length and simply supported edges under pressure and thermal loads is presented. Material properties of the FG plate is assumed to be graded in the thickness direction according to a simple exponent-law distribution in terms of the volume fractions of the constituents. The partial differential equations are reduced to the ordinary one by expanding the state and induced variables into double Fourier series with respect to the in-plane coordinates and then are solved by state-space method.

## 2. Basic equations

A functionally graded rectangular plate with length $a$, width $b$ and thickness $h$, as shown in Fig. 1, is considered. The plate is simply supported at all edges and subjected to uniform steady-state temperature loads, $T_h$ and uniform pressure, $P_0$ on the top surface and zero temperature on the bottom surface as well as on the four ends surfaces. The FG plate is transversely isotropic with constant Poisson's ratio, $v$ and the other material thermo-elastic properties with the exponential distribution across the thickness, as follow:

$$E = E_0 e^{\beta_1 z} \tag{1a}$$

$$\alpha = \alpha_0 e^{\beta_2 z} \tag{1b}$$

$$\lambda = \lambda_0 e^{\beta_3 z} \tag{1c}$$

where $\beta_1, \beta_2, \beta_3$ are material constants which $\beta_1 = \beta_2 = \beta_3 = 0$ the FG plate reduce to a homogeneous plate.

Thermo-elastic constitutive relations of FG plate in term of displacements are as follows:

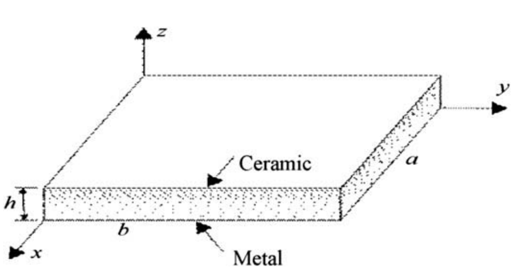


**Fig. 1.** Functionally graded plate with coordinate system.

$$\begin{aligned}
\sigma_x &= \frac{E}{(1+v)(1-2v)}\left[(1-v)u_{,x} + v v_{,y} + v w_{,z}\right] - \frac{\alpha E}{1-2v}T \\
\sigma_y &= \frac{E}{(1+v)(1-2v)}\left[v u_{,x} + (1-v) v_{,y} + v w_{,z}\right] - \frac{\alpha E}{1-2v}T \\
\sigma_z &= \frac{E}{(1+v)(1-2v)}\left[v u_{,x} + v v_{,y} + (1-v) w_{,z}\right] - \frac{\alpha E}{1-2v}T \\
\tau_{xy} &= \frac{E}{2(1+v)}(u_{,y} + v_{,x}) \\
\tau_{xz} &= \frac{E}{2(1+v)}(u_{,z} + w_{,x}) \\
\tau_{yz} &= \frac{E}{2(1+v)}(v_{,z} + w_{,y})
\end{aligned} \tag{2}$$

The stress components should satisfy the following equilibrium equations:

$$\begin{aligned}
\sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} &= 0 \\
\tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} &= 0 \\
\tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} &= 0
\end{aligned} \tag{3}$$

The steady state three-dimensional heat conduction equation is taken in the following form:

$$\lambda_x \frac{\partial^2 T}{\partial x^2} + \lambda_y \frac{\partial^2 T}{\partial y^2} + \frac{\partial \lambda_z}{\partial z}\frac{\partial T}{\partial z} + \lambda_z \frac{\partial^2 T}{\partial z^2} = 0 \tag{4}$$

The thermal, displacements and stresses boundary conditions are

$$T(0, y, z) = T(a, y, z) = 0 \tag{5a}$$

$$T(x, 0, z) = T(x, b, z) = 0 \tag{5b}$$

$$T(x, y, 0) = T_i \tag{5c}$$

$$T(x, y, h) = T_0 \tag{5d}$$

$$\sigma_z = -P_0, \tau_{xz} = \tau_{yz} = 0 \quad \text{at } z = h \tag{6a}$$

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0 \quad \text{at } z = 0 \tag{6b}$$

$$\sigma_x = v = w = 0 \quad \text{at } x = 0, a \tag{6c}$$

$$\sigma_y = u = w = 0 \quad \text{at } y = 0, b \tag{6d}$$

## 3. Analytical solution for temperature field

The assumed solution to Eq. (4) satisfying the temperature boundary conditions, (6a) and (6b), is

$$T = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{mn} e^{-\beta_2 z} \sin(p_m x) \sin(p_n y) \tag{7}$$

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