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Dynamic response of thick laminated annular sector plates subjected to moving load

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ABSTRACT

The dynamic response of thick laminated annular sector plates with simply supported radial edges subjected to a radially distributed line load, which moves along the circumferential direction, is studied. A three-dimensional hybrid method composed of series solution, the layerwise theory and the differential quadrature method in conjunction with the finite difference method is employed. The fast rate of convergence and high accuracy of the method are demonstrated through different examples. Additionally, as a limit case, the out-of-plane dynamic responses of circular curved beams is obtained and compared with those of an unconstrained higher order shear deformation curved beam theory, which is formulated here. Then, the effects of different parameters such as the sector angle, thickness-to-outer radius ratio, ply lay out and the load velocity on the out-of-plane response of the symmetric and antisymmetric cross-ply laminated sector plates are investigated. The results can be used as benchmark solutions for future works.

1. Introduction

Laminated annular sector plates have found wide applications as structural members in aerospace, marine and other industries. In the limit case that the width of the annular sector plate is small in comparison with its outer radius, it can be treated as a circular curved beam. There are a lot of research works on the free vibration analyses of isotropic and laminated sector plates, some recent works of which are cited in Refs. [1–7]. However, to the best of authors' knowledge, the dynamic analysis of such a structural elements under the action of moving load is limited to out-of-plane response of curved beams [8,9].

To fill this gap, and as a continuation of the previous work of the first author [7], the dynamic response of symmetric and antisymmetric laminated annular sector plates subjected to a radially distributed line load, which moves along the circumferential direction, is studied here. For this purpose, an approach benefits the capability of the layerwise theory to accurately model the variation of material properties across the thickness [10–12], and the differential quadrature method (DQM) as an efficient and accurate numerical method to discretize the variable coefficients differential equations in the radial direction [7,13–17] is adopted. The temporal domain is discretized using the finite difference method. The formulation is based on the three-dimensional linear elasticity theory. After studying the convergence behavior of the method, as a limit case, the out-of-plane responses of laminated circular curved beams are obtained and comparison studies with those of an unconstraint higher shear deformation theory (HSDT) are made. Additionally, the static analyses of annular sector plates are investigated and comparison studies with available results are carried out. Finally, the effects of different parameters such as the sector angle, thickness-to-outer radius ratio, ply lay out and the load velocity on the out-of-plane response of the symmetric and antisymmetric cross-ply laminated sector plates are investigated.

2. Mathematical modeling

Consider a laminated thick annular sector plates composed of N_L perfectly bonded orthotropic layers of width b, total thickness h, sector angle θ_0, inner radius R_i and outer radius R_o (Fig. 1). Each lamina is considered to be cylindrically orthotropic with the fiber orientation being either in the radial or circumferential direction. The fiber angle is measured from the radial axis.

Based on the three-dimensional layerwise theory, the laminated plate is divided into $N_m(\geqslant N_L)$ mathematical layers. In each lamina, the displacement components are approximated in a similar manner as the one-dimensional finite element method. In the present study, one-dimensional Lagrange interpolation functions are used in each mathematical layer and hence the global interpolation function φ_i can easily be obtained [18]. Then, for the sector plates with simply supported radial edges, the displacement components at an arbitrary material point of the plate can be expressed as

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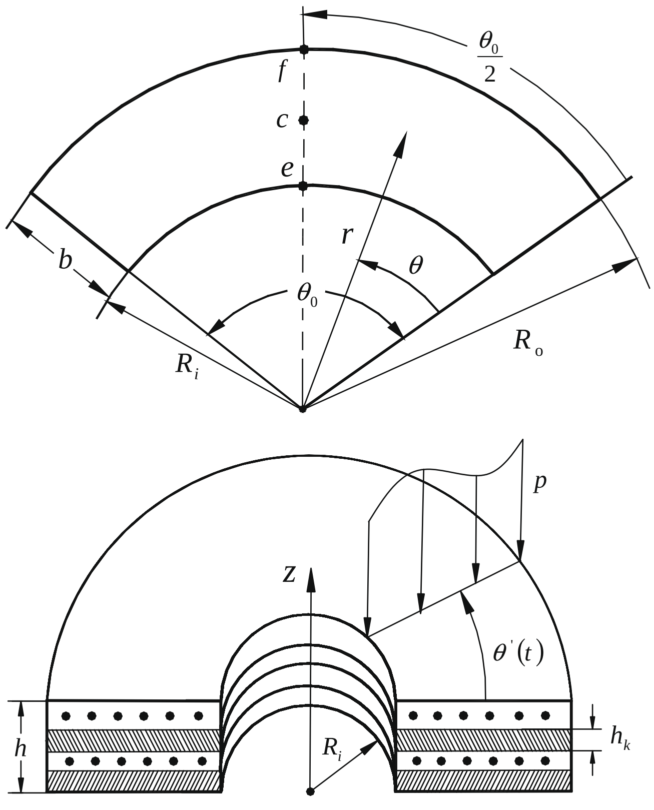

Fig. 1. An arbitrary laminated thick annular sector plate.

$$\begin{Bmatrix} u(r,\theta,z,t) \\ v(r,\theta,z,t) \\ w(r,\theta,z,t) \end{Bmatrix} = \sum_{m=1}^{N_\theta}\sum_{i=1}^{N_z} \begin{Bmatrix} U_{im}(r,t)\sin(\beta_m\theta) \\ V_{im}(r,t)\cos(\beta_m\theta) \\ W_{im}(r,t)\sin(\beta_m\theta) \end{Bmatrix} \varphi_i(z) \tag{1}$$

where $\beta_m = m\pi/\theta_0$; u, v and w are the displacement components along the radial (r), the circumferential (θ) and the thickness (z) directions, respectively; φ_i denotes the global interpolation function of the node 'i' (defined by $z = z_i$) in the z-direction and its explicit form is given in Ref. [18]; U_{im}, V_{im} and W_{im} represent the displacement functions of node 'i' in the r, θ- and z-directions, respectively; N_z stands for the total number of nodes through-the-thickness of the plate [7,18].

The through-the-thickness and the circumferential discretized equations of motion at each layerwise node 'i' can be obtained by using the Hamilton's principle, which in this case turns into,

$$\int_{t_1}^{t_2}\int_0^h\int_0^{\theta_0}\int_{R_i}^{R_o}\Big\{\sigma_{rr}\delta\varepsilon_{rr}+\sigma_{\theta\theta}\delta\varepsilon_{\theta\theta}+\sigma_{zz}\delta\varepsilon_{zz}+\sigma_{r\theta}\delta\gamma_{r\theta}+\sigma_{rz}\delta\gamma_{rz} + \sigma_{\theta z}\delta\gamma_{\theta z} - \rho^{(k)}\left[\frac{\partial u}{\partial t}\delta\left(\frac{\partial u}{\partial t}\right)+\frac{\partial v}{\partial t}\delta\left(\frac{\partial v}{\partial t}\right)+\frac{\partial w}{\partial t}\delta\left(\frac{\partial w}{\partial t}\right)\right] - p(r,\theta,t)\delta(z-h/2)\delta w\Big\} r\,dr\,d\theta\,dz\,dt = 0 \tag{2}$$

where $\rho^{(k)}$ is the mass density of the kth layer; t is the temporal variable; t_1 and t_2 are two arbitrary times; $p(r, \theta, t)$ is the transverse load per unit area on the upper surface of the plate; δ is the delta function; ε_{ii} and γ_{ij} ($i, j = r, \theta, z$ with $i \neq j$) are the normal and the shear components of the strain tensor, respectively; and σ_{ij} ($i, j = r, \theta, z$) are the stress tensor components.

Substituting the three-dimensional strain–displacement and the stress–strain relations [7] together with Eq. (1) into Eq. (2), yields the equations of motion and the related boundary conditions at each node 'i' with $i = 1, 2, \ldots, N_z$ as,

Equations of motion:
δU_{im}:

$$\sum_{j=1}^{N_z} A_{11}^{ij}\left(r\frac{\partial^2 U_{jm}}{\partial r^2}+\frac{\partial U_{jm}}{\partial r}\right) - \sum_{j=1}^{N_z}\left[\frac{(A_{22}^{ij}+\beta_m^2 A_{66}^{ij})}{r} + D_{55}^{ij} r\right]U_{jm} + \left(\frac{\beta_m}{r}\right)\sum_{j=1}^{N_z}\left(A_{22}^{ij}+A_{66}^{ij}\right)V_{jm} - \beta_m\sum_{j=1}^{N_z}\left(A_{12}^{ij}+A_{66}^{ij}\right)\frac{\partial V_{jm}}{\partial r} - r\sum_{j=1}^{N_z}\left(B_{55}^{ij}-B_{13}^{ji}\right)\frac{\partial W_{jm}}{\partial r} - \sum_{j=1}^{N_z}\left(B_{23}^{ji}-B_{13}^{ji}\right)W_j - r\sum_{j=1}^{N_z} I^{ij}\frac{\partial^2 U_{jm}}{\partial t^2} = 0 \tag{3}$$

δV_{im}:

$$\beta_m\sum_{j=1}^{N_z}(A_{12}^{ij}+A_{66}^{ij})\frac{\partial U_{jm}}{\partial r} + \left(\frac{\beta_m}{r}\right)\sum_{j=1}^{N_z}(A_{22}^{ij}+A_{66}^{ij})U_{jm} + \sum_{j=1}^{N_z}A_{66}^{ij}\left(r\frac{\partial^2 V_{jm}}{\partial r^2}+\frac{\partial V_{jm}}{\partial r}\right) - \sum_{j=1}^{N_z}\left(\frac{\beta_m^2 A_{22}^{ij}}{r}+D_{44}^{ij}r+\frac{A_{66}^{ij}}{r}\right)V_{jm} - \beta_m\sum_{j=1}^{N_z}\left(B_{44}^{ji}-B_{23}^{ij}\right)W_{jm} - r\sum_{j=1}^{N_z}I^{ij}\frac{\partial^2 V_{jm}}{\partial t^2} = 0 \tag{4}$$

δW_{im}:

$$r\sum_{j=1}^{N_z}\left(B_{13}^{ij}-B_{55}^{ji}\right)\frac{\partial U_{jm}}{\partial r} + \sum_{j=1}^{N_z}(B_{23}^{ij}-B_{55}^{ji})U_{jm} - \beta_m\sum_{j=1}^{N_z}\left(B_{23}^{ij}-B_{44}^{ji}\right)V_{jm} - \sum_{j=1}^{N_z}A_{55}^{ij}\left(r\frac{\partial^2 W_{jm}}{\partial r^2}+\frac{\partial W_{jm}}{\partial r}\right) + \sum_{j=1}^{N_z}\left(\frac{\beta_m^2 A_{44}^{ij}}{r}+rD_{33}^{ij}\right)W_{jm} + r\sum_{j=1}^{N_z}I^{ij}\frac{\partial^2 W_{jm}}{\partial t^2} = rp_m(r,t)\delta_{iN_z} \tag{5}$$

Boundary conditions on the surfaces $r = R_i$ and R_o:

$$\text{Either}\quad U_{im}(r,t)=0 \quad\text{or}\quad r\sum_{j=1}^{N_z}A_{11}^{ij}\frac{\partial U_{jm}}{\partial r} + \sum_{j=1}^{N_z}A_{12}^{ij}(U_{jm}-\beta_m V_{jm}) + r\sum_{j=1}^{N_z}B_{13}^{ji}W_{jm} = 0 \tag{6}$$

$$\text{Either}\quad V_{im}(r,t)=0 \quad\text{or}\quad \sum_{j=1}^{N_z}A_{66}^{ij}\left(\beta_m U_{jm} - V_{jm} + r\frac{\partial V_{jm}}{\partial r}\right) = 0 \tag{7}$$

$$\text{Either}\quad W_{im}(r,t)=0 \quad\text{or}\quad \sum_{j=1}^{N_z}\left(B_{55}^{ji}U_{jm} + A_{55}^{ij}\frac{\partial W_{jm}}{\partial r}\right) = 0 \tag{8}$$

where $p_m(r,t) = \left(\frac{2}{\theta_0}\right)\int_0^{\theta_0} p(r,\theta,t)\sin(\beta_m\theta)d\theta$ and δ_{ij} is the Kronecker delta. The stiffness and inertia coefficients are obtained by exact integrations from the following expressions,

$$A_{mn}^{ij} = \int_0^h C_{mn}^{(k)}\varphi_i\varphi_j dz, \quad B_{mn}^{ij} = \int_0^h C_{mn}^{(k)}\varphi_j\frac{d\varphi_i}{dz}dz, \quad D_{mn}^{ij} = \int_0^h C_{mn}^{(k)}\frac{d\varphi_i}{dz}\frac{d\varphi_j}{dz}dz, \quad I^{ij} = \int_0^h \rho^{(k)}\varphi_i\varphi_j dz \tag{9}$$

where $C_{mn}^{(k)}$ are the elastic coefficients of the kth layer [18].

Since it is very hard, if it not impossible, to obtain an exact solution for the coupled and variable coefficients differential Eqs. (3)–(5), here the differential quadrature method as an efficient, simple and accurate numerical method is applied here [7,13–17].