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Composite Structures 83 (2008) 48–60

COMPOSITE STRUCTURES

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Free vibration and buckling analyses of functionally graded beams with edge cracks

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Available online 21 March 2007

Abstract

This paper presents a theoretical investigation in free vibration and elastic buckling of beams made of functionally graded materials (FGMs) containing open edge cracks by using Bernoulli–Euler beam theory and the rotational spring model. It is assumed that the material properties vary along the beam thickness only according to exponential distributions. Analytical solutions of the natural frequencies, critical buckling load, and the corresponding mode shapes are obtained for cracked FGM beams with clamped–free, hinged–hinged, and clamped–clamped end supports. A detailed parametric study is conducted to show the influences of the location and total number of cracks, material properties, slenderness ratio, and end supports on the flexural vibration and buckling characteristics of cracked FGM beams.

Keywords: Functionally graded materials; Beam; Open edge crack; Vibration; Buckling

1. Introduction

Cracks in a structural element in the form of initial defects within the material or caused by fatigue or stress concentration can reduce the natural frequencies and change the vibration mode shapes due to the local flexibility introduced by the crack. Understanding the dynamic characteristics of cracked structures is of prime importance in structural health monitoring and non-destructive damage evaluation because the predicted vibration data can be used to detect, locate, and quantify the extent of the cracks or damages in a structure. A large number of investigations in the free vibration of cracked beams are available in open literature, see, for example, those by Dimarogonas [1], Gounaris et al. [2], Chondros et al. [3], Shifrin and Ruotolo [4], Binici [5]. Basically, the theoretical modeling techniques can be grouped into "continuous" models and "lumped flexibility" models. The first category uses a continuous, one-dimensional cracked beam model through variational principles. The finite element method, Galerkin method, and Rayleigh–Ritz method are the commonly used numerical approaches in this category. The second category represents the presence of a crack and the reduction in the beam bending stiffness by means of a line spring, and the equivalent lumped stiffness is determined through fracture mechanics analysis, starting from the knowledge of stress intensity factor (SIF). Among those in this category, Yokoyama and Chen [6] obtained the vibration characteristics of a Bernoulli–Euler beam with a single edge crack based on a modified line-spring model. Zheng and Fan [7] computed the natural frequencies of a Timoshenko beam with an arbitrary number of transverse open cracks. A modified Fourier series solution technique was developed. By using the differential quadrature method and the spring model, Hsu [8] studied the flexural vibration and dynamic response of edge-cracked Bernoulli–Euler beams resting on an elastic foundation and subjected to an axial loading and lateral excitation force. El Bikri et al. [9] investigated the geometrically non-linear free

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0263-8223/$ - see front matter
doi:10.1016/j.compstruct.2007.03.006

vibrations of a clamped–clamped beam containing an open crack. A semi-analytical model based on an extension of the Rayleigh–Ritz method was used in their study.

Another important issue deserving special attention in a cracked structure is the weakened buckling capacity due to the existence of cracks. Wang [10] gave a comprehensive study on the stability of a cracked beam subjected to a follower compressive load. Zhou and Huang [11] presented a closed form solution of the maximum deflection for cracked columns with rectangular cross-sections and studied the elastic buckling problem analytically. A repair technique using a piezoelectric patch that produce a local moment to counteract the loss of bending stiffness to restore the buckling load-carry capacity of a cracked column has also been proposed recently [12].

Functionally graded materials (FGMs), a novel class of macroscopically inhomogeneous composites with spatially continuous material properties, have attracted considerable research efforts over the past few years due to their increasing applications in many engineering sectors. Numerous studies have been conducted on FGM beams, plates and shell structures, dealing with a variety of subjects such as thermal elasticity [13–16], fracture analysis [17–20], static bending [21–24], free vibration and dynamic response [25–29], buckling and postbuckling [30–33], piezo-thermo-elastic behavior [34–36], and so on. Literature review shows that although there are quite a few papers presenting crack and fracture analyses of FGM structures, no previous work investigating the vibration and buckling behavior of cracked FGM structures has been reported.

The objective of this paper is to study the free vibration and elastic buckling of slender FGM beams with open edge cracks to gain an insight into the effects of material property distribution, the total number and location of cracks, the slenderness ratio and boundary condition on the vibrational and buckling characteristics of edge-cracked FGM beams. The classical Bernoulli–Euler beam theory and the rotational spring model are used in the present study. Comprehensive numerical results are obtained analytically for beams with clamped–free, hinged–hinged, and clamped–clamped boundary conditions.

2. The rotational spring model

Consider an FGM beam of length L and thickness h, containing an edge crack of depth a located at a distance L_1 from the left end as shown in Fig. 1. The shear modulus ν, Young's modulus E, and mass density ρ of the beam vary in the thickness direction only and follow the exponential distributions below

$$\nu(z) = \nu_0 e^{\beta z}, \quad E(z) = E_0 e^{\beta z}, \quad \rho(z) = \rho_0 e^{\beta z} \tag{1}$$

where ν_0, E_0, and ρ_0 are the values of the shear modulus, Young's modulus, and mass density at the midplane ($z = 0$) of the beam. β is a constant defining the material property variation along the thickness direction, and $\beta = 0$ corresponds to an isotropic homogeneous beam.

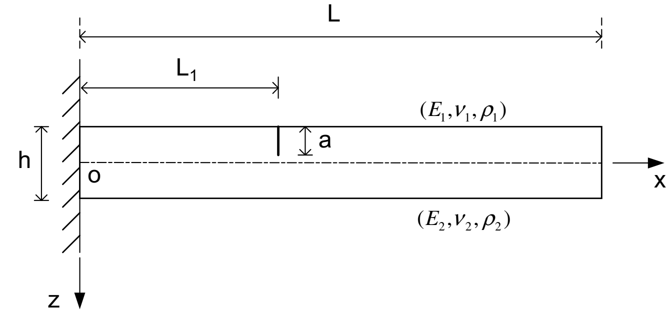

Fig. 1. An FGM beam with an open edge crack.

Poisson's ratio μ is taken as a constant since its influence on the stress intensity factors (SIF) is quite limited [18].

It is assumed that the crack is perpendicular to the beam surface and always remains open. Based on the well-accepted rotational-spring model, the cracked beam can be treated as two sub-beams connected by an elastic rotational spring at the cracked section which has no mass and no length. The bending stiffness of the cracked section k_T is related to the flexibility G by

$$k_T = \frac{1}{G} \tag{2}$$

From Broek's approximation [37], the flexibility of the beam G due to the presence of the crack can be derived as

$$\frac{1-\mu^2}{E(z)} K_I = \frac{M_I^2}{2} \frac{dG}{da} \tag{3}$$

where M_I is the bending moment at the cracked section, K_I is the stress intensity factor (SIF) under mode I loading and is a function of the geometry, the loading, and the material properties as well. For an FGM strip with an open edge crack under bending, the expression of SIF was derived by Erdogan and Wu [18] as

$$K_I = -\frac{4\sqrt{a}\nu(z)}{1+\bar{\mu}} \sum_{n=0}^{\infty} \alpha_n T_n\left(\frac{2z-a}{a}\right) \tag{4}$$

where $\bar{\mu} = (3 - 4\mu)$ for plane strain problem, T_n is the Chebyshev polynomial of the first kind. The constants α_n are determined by evaluating the boundary integrals using Gaussian quadrature and then solving the resulting functional equation by collocation method. The detailed solution process can be found in Erdogan and Wu's paper [18].

3. Free vibration analysis

Based on the Kirchhoff–Love hypothesis, the displacements parallel to the x- and z-axes of an arbitrary point

Table 1
Fundamental frequency ratio ω_1/ω_{10} of an isotropic homogenous clamped–free beam

	$L_1/L = 0.2$	$L_1/L = 0.4$	$L_1/L = 0.6$
Present	0.97833	0.99107	0.99776
Yokoyama and Chen [6]	0.94101	0.96667	0.99583