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S-shaped mode in the lower and upper bounds of the buckling of composite beams with two equal delaminations

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Abstract

In this paper, lower and upper bounds of the buckling load of a composite beam with two equal delaminations are obtained by developing analytical models. The characteristic equations governing the delamination buckling are derived by using Euler–Bernoulli beam theory, performing proper linearization and by imposing the appropriate continuity and boundary conditions. The effects of the differential stretching and the bending-extension coupling are considered. The accuracy of the models is verified by comparing results with previously published data and a separately carried out finite element analysis. The effects of the dimensions and locations of the two equal delaminations on the lower and upper bounds of the buckling load are investigated in detail. The lower and upper bounds of the buckling load strongly depend on the sizes and locations of the two equal delaminations. For certain lengths and thicknesswise locations of the two equal delaminations, S-shaped buckling mode strongly influences the buckling behavior for both the upper and lower bounds of the buckling load. The lower and upper bounds of the buckling load will be useful to gauge the working range of the bridging and give guidelines for practical applications.

Keywords: Buckling; Delamination; Composites; Finite element analysis; Mode shapes

1. Introduction

Composite structures are designed to withstand extreme loading conditions during services which include both static and dynamic loads. The life of the structure is determined by a failure criterion defining an acceptable level of stiffness or strength. There are several failure mechanisms that contribute to the property degradation of the composite structures, such as broken fibres, fibre debonding [1], delaminations [2] and micro cracks of the matrix [3]. For multilayer composite materials delaminations are one of the most common failure modes, which can be initiated at the interlaminar region by impact loading, fatigue and/or poor manufacturing process. Under compression, a delaminated composite laminate may buckle and possibly undergo propagation of the delaminations. This propagation can lead to lowering of the buckling load of the laminate and lead to an unexpected structural failure at loads below the design level. The delamination buckling analyses become further intricate as the beams are made of different material or hybrid composites, and the occurrence of the delaminations vary in number (single or multiple), type (enveloping, non-overlapping, equal, etc.), locations (spanwise or thicknesswise) and shapes (embedded or through-width).

With the increasing use of the composite laminates, the compression behavior of delaminated layer structures has received considerable attention in the recent years. One and two-dimensional mathematical models [4–12], finite element analyses [13–19], experimental investigations [20] have been carried out to study delamination buckling of beams and plates. More recently, the current authors have studied the delamination buckling of two- and three-layer delaminated beams in terms of newly defined nondimensionalized axial and bending stiffnesses and effective-slenderness ratio [21,22].

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Analytical methods to obtain lower and upper bounds of the buckling load of composite beams with two equal delaminations, by considering the effects of the differential stretching and the bending-extension coupling have not yet been reported. For the case of a beam with a single delamination, it was found that these two bounds will be useful to gauge the working range of the bridging [23] and give guidelines for practical applications [24]. The research reported in this paper has three objectives: first, to develop mathematical models that can be used to obtain lower and upper bounds of the buckling load of composite beams with two equal delaminations. Secondly, to verify the models by comparing the results obtained using the finite element analysis and the literature. Finally, to use these models to investigate the buckling of composite beams with two equal delaminations. The paper is organized as follows: Section 2 involves the development of the analytical models employed to obtain the lower and upper bounds of the buckling load. Section 3 contains the finite element analysis. Results are discussed in Section 4 and conclusions are presented in Section 5.

2. Formulation

The geometry of a composite beam with two equal delaminations is illustrated in Fig. 1a. The pre-existing through width two equal delaminations are of length a, denoted as delamination-I (located nearer to the top surface of the beam) and delamination-II (located further deep inside) and are located at an offset distance of d from the center of the beam, respectively. The length of the beam is L and thickness is H. Considering a one-dimensional through the width delaminations conveniently defines a unit width of the composite beam. Both ends of the beam are assumed to be clamped and an external load P is applied along the center of the beam. Due to the presence of the two equal delaminations, the composite beam is analyzed as five interconnected beams, i.e., virgin-beams 1 and 5 (non-delaminated portion), and sub-beams 2, 3 and 4 (delaminated portion) as shown in Fig. 1a and b. The virgin and sub-beams have thicknesses of H_i and lengths of L_i ($i = 1$–5), respectively. In Fig. 1a, notations A and F denote the clamped ends and B, C, D and E denote the delamination ends. The coordinate axes for the sub-beams are shown in Fig. 1a and x is mea-

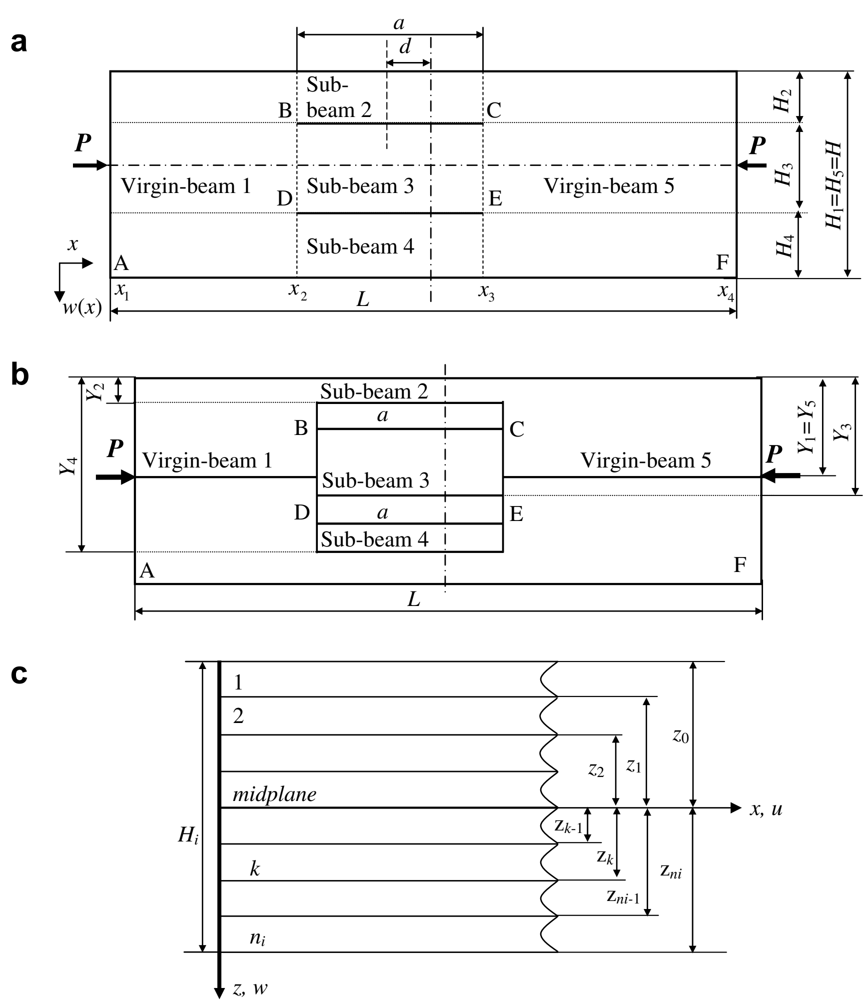

Fig. 1. A composite beam with two equal delaminations: (a) under axial compressive loads; (b) equivalent beam model (shown with neutral axes); (c) i^{th} beam laminated.