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Mixed mode fracture in an inclined center crack repaired by composite patching

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Abstract

The use of bonded composite patches is among favorite reinforcement methods for repairing cracked structures. Although extensive research studies deal with the safety evaluation and life prediction of repaired components, most of these investigations are concentrated on pure mode I loading (or opening mode of crack deformation). Meanwhile in many engineering structures, cracked components are subjected to mixed mode I/II. In this paper the effects of composite patching are investigated on fracture behavior of an inclined center crack, under different combination of modes I and mode II loading conditions. A three-dimensional finite element model of the single-sided repaired specimen is used to study the effect of composite patching on the crack tip parameters (K_I, K_{II} and T-stress). Then the fracture initiation angle and the fracture strength of repaired crack are predicted by using the generalized maximum tangential stress criterion. The effects of composite laminate configuration and adhesive properties on the performance of bonded composite patch are also investigated herein. It is shown that parameters like mode mixing, patch thickness and properties of film adhesive can influence significantly the crack tip parameters and hence the fracture strength of the repaired specimens under mixed mode loading conditions.

Keywords: Composite patch; Brittle fracture; Generalized MTS criterion; Stress intensity factor; T-stress

1. Introduction

Due to various benefits of composite materials such as light weight, high strength and their excellent formability, the externally bonded composite patches have been proved to be a preferable method of repairing flaws and cracks in various engineering structures. This repairing method initially was investigated in Australia in early 1970s [1–3] and later in USA in 1980s [4]. Later on, considerable studies have been performed to develop this technology by various experimental and numerical methods. Among them Jones and Callinan [5], Bachir et al. [6], Chung and Yang [7], Kumar and Ripudaman [8] and Ayatollahi and Hashemi [9] have used the finite element method to investigate the effect of composite patching on the stress intensity factor of crack, as an important measure for analyzing the performance of composite reinforcement technique. Most of the previous studies are limited to the case of pure mode I. However, the cracks existing in engineering structures very often experience a complicated loading history. For example, under a combined shear and tension, the cracks are subjected to mixed mode I/II loading and the influence of composite patch and its design parameters should be investigated on the crack tip parameters, i.e. K_I, K_{II} and T-stress.

The aim of this paper is to investigate the effects of composite reinforcement on the fracture strength of a crack under mixed mode I/II conditions. For this purpose a three-dimensional finite element model of the centre cracked plate is provided in *Abaqus* [10]. Prior to loading the specimen unidirectionally, the cracked specimen is reinforced by bonding a graphite/epoxy composite patch on its one single side. The crack tip parameters, K_I, K_{II} and T-stress are determined for different crack inclination angles. By substituting these parameters into the equations of *generalized maximum tensile stress* (*GMTS*) *criterion*, the

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Nomenclature

a	semi-crack length	t_a	film adhesive thickness
K_{Ic}	mode I fracture toughness	t_r	composite patch thickness
K_{eff}	effective stress intensity factor	u_r, u_θ	displacement components
K_I	mode I stress intensity factor	W_p	width of cracked plate
K_{II}	mode II stress intensity factor	W_r	width of composite patch
G_a	adhesive shear modulus	β	crack inclination angle
r_c	critical distance from the crack tip	θ_0	fracture initiation angle
r, θ	crack tip coordinates	σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$	stress components
T	non-singular stress term	$\sigma_{\theta\theta c}$	critical value of tangential stress

fracture initiation angle and the strength of repaired crack are obtained under various mixed mode I/II loading conditions. Furthermore, the effect of patching design variables like thickness and width of composite laminate and thickness and shear modulus of film adhesive are investigated individually.

2. Fracture criterion

For a crack in an isotropic elastic material subjected to mixed mode I/II loading, the series expansion of stress field are written from William's solution as [11]

$$\sigma_{rr} = \frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[K_I\left(1+\sin^2\frac{\theta}{2}\right) - K_{II}\left(\frac{3}{2}\sin\theta - 2\tan\frac{\theta}{2}\right)\right] + T\cos^2\theta + O(r^{1/2}) \tag{1}$$

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[K_I\cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] + T\sin^2\theta + O(r^{1/2}) \tag{2}$$

$$\sigma_{r\theta} = \frac{1}{2\sqrt{2\pi r}}\cos\frac{\theta}{2}[K_I\sin\theta + K_{II}(3\cos\theta - 1)] - T\sin\theta\cos\theta + O(r^{1/2}) \tag{3}$$

where r and θ are the conventional crack tip coordinates (see Fig. 1) and K_I and K_{II} are the mode I and mode II stress intensity factors (SIF), respectively. The term T is a constant stress parallel to the crack and its magnitude depends on the geometry and the load applied to the cracked structure. The higher order terms $O(r^{1/2})$ can be neglected near the crack tip. Numerous studies have shown that the T-stress has an important role in strength and stability analysis of cracks. In elastic–plastic problems, it is now well recognized that the T-stress influences the size and shape of the plastic zone significantly [12]. Meanwhile, the T-stress has also an important effect on brittle fracture under mixed mode I/II loading, such that a combination of singular term (characterized by K_I and K_{II}) and T-stress controls the crack extension [13–15]. Indeed, fracture of solids under mixed mode loading has drawn attention due to its close proximity to realistic conditions in engineering structures. There are several criteria for prediction of brittle fracture under mixed mode loading. The maximum tensile stress (MTS) criterion [16], the maximum energy release rate criterion [17] and the minimum strain energy density criterion [18] are to name a few. However, many researchers have reported that these criteria do not comply with experimental results for several mixed mode test specimens because they take into account only the singular term of stress field around the crack tip [13,15,19,20]. Smith et al. [15] modified the maximum tensile stress criterion by taking into account the effect of T-stress in addition to singular terms in the tangential stress around the crack tip and could provide more consistency between theoretical and experimental results. According to this criterion, known as generalized maximum tangential stress criterion, crack growth initiates from the crack tip along the direction of maximum tangential stress

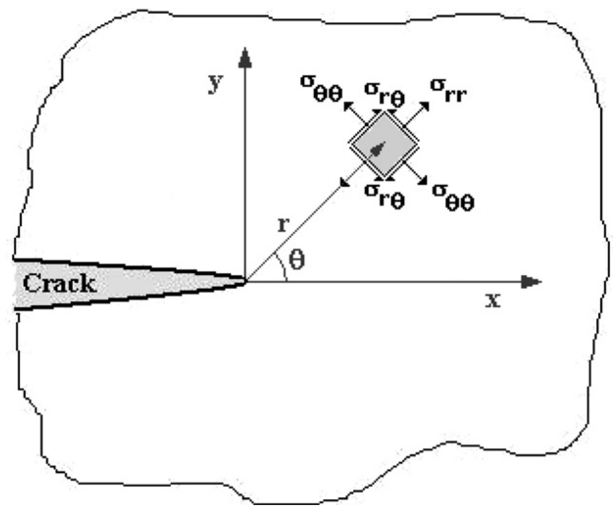

Fig. 1. Definition of conventional coordinate frames near the crack tip.

$$\frac{\partial\sigma_{\theta\theta}}{\partial\theta} = 0, \quad \frac{\partial^2\sigma_{\theta\theta}}{\partial\theta^2} < 0 \Rightarrow \theta = \theta_0 \tag{4}$$