Composite Structures 90 (2009) 100–103

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Technical Note

Free vibration analysis of simply supported composite laminated panels

M. Ganapathi *, Amit Kalyani, Bhaskar Mondal, T. Prakash

JFWTC, General Electric India Technology Centre, Bangalore – 560 066, India

ARTICLE INFO

Article history:
Available online 15 February 2009

Keywords:
Vibrations
Anisotropic
Analytical
Sandwich
Laminates
Curved panel

ABSTRACT

In this paper, the free vibrations characteristics of simply supported anisotropic composite laminates are investigated using analytical approach. The formulation is based on the first-order shear deformation theory and the shear correction factors employed are based on energy consideration that depends on the lay-up as well as material properties. The governing equations are obtained using energy method. For this purpose, a displacement model with a combination of sine and cosine functions in the form of double Fourier series is taken. The effectiveness of the integrated formulation is tested for global behaviors considering examples related to multi-layered laminates having various values for curvatures, thickness ratio, and lay-up for which solutions are available.

1. Introduction

There is a renewed interest in the use of laminated/sandwich construction for future marine systems, large transport aircraft, high power wind-turbine, etc. Hence, there is an increased activity among engineers while modeling the behaviors of such structures under dynamic situations, leading to accurate solution and considerable savings in computational time.

It can be seen from the literature that the amount of work carried out on the vibration characteristics of isotropic plates and composite laminates are exhaustive. Some of the important contributions are briefly mentioned here. Crawley and Dugundji [1] studied the free vibration characteristics of a cantilever configuration based on a partial Ritz analysis, which reduces the problem to a set of uncoupled ordinary differential equations. Iyengar and Umaretiya [2] considered hybrid composite plates with two opposite edges simply supported and the other two edges clamped. Noor [3] presented the three dimensional elasticity solutions for isotropic, orthotropic and anisotropic laminated composite plates which serve as bench mark solutions for comparison by many researchers. Further, within the scope of linear theory, free vibration behavior of laminated curved panels is also studied in Refs. [4–9]. In most of these studies, closed-form solutions could be obtained for antisymmetric cross-ply laminates. Fortier [4] used the Rayleigh–Ritz method for the analysis of some angle-ply laminated curved panels without shear deformation. Librescu et al. [5] developed a simple shear-deformable theory for doubly curved shallow cross-ply composite shells and used state-space concepts in conjunction with the Levy method to evaluate the static and dynamic response of panels for various boundary conditions. Chandrashekhara [6] studied the free vibration characteristics of curved panels using an isoparametric doubly curved quadrilateral shear flexible element. Barai and Durvasula [7] investigated the vibration and buckling of curved plates, made of hybrid laminated composite materials, based on analytical approach whereas Ferreiraa et al. [8] employed layer-wise theory coupled with collocation method to study static and vibration behavior of sandwich plates. The mixed formulation leading equivalent single layer model was highlighted by Rao and Desai [10]. Kant and Swaminathan [11], and Ganapathi and Makhecha [12] have presented solutions for laminated composite laminates based on higher-order shear deformation theories. It may be concluded from the available literature that the sandwich construction work are mainly dealt with either zigzag deformation pattern or a three-layered model coupled with the first-order shear deformation. However, higher-order models that involve additional displacement fields may be based on either an equivalent single layer theory or a discrete layer approach. Furthermore, they are computationally expensive in the sense that the number of unknown to be solved is high compared to that of the first-order shear deformation formulation.

Here, the vibration analysis of composite laminates is examined based on first-order shear deformation theory. The shear correction factors evaluated based on energy consideration, as outlined in Ref. [13] are introduced in the present formulation. These factors depend on the layers properties and stacking sequence, and in turn will be different from 5/6 as used in the literature. The accuracy of the integrated formulation is examined for frequencies of anisotropic sandwich panels, considering problems for which solutions are available in the literature.

* Corresponding author. Present address: Engineering Solutions (Aerospace Engineering), ITC-5, II Floor (B-wing), Satyam Computers Services Ltd., Electronic City (Phase-2), Bangalore – 560 100, India.

E-mail addresses: Ganapathi_Manickam@Satyam.com, mganapathi@rediffmail.com (M. Ganapathi).

0263-8223/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2009.02.003

2. Formulation

A rectangular shallow curved composite panel (length a, width b, and thickness h) is considered with the coordinates x, y along the in-plane directions, z along the thickness direction. For the present study, the equation of the mid-plane of the curved panel with curvature in x and y direction [14] is considered as

$$\frac{z_p}{\delta} = 2 - \frac{[x-(a/2)]^2}{(a/2)^2} - \frac{[y-(b/2)]^2}{(b/2)^2} \quad (1)$$

Here, 2δ represents the maximum initial height. For a shallow curved panel the value $(2\delta/a)$ is small. The mid-plane radii of curvature are given by:

$$R_{ij} = 1/z_{p,ij} \quad (i,j = x,y) \quad (2)$$

where the subscript comma denotes the partial derivative with respect to the spatial coordinate succeeding it.

The displacements u, v, w at a point (x,y,z) in the shell (Fig. 1) from the medium surface are expressed as functions of mid-plane displacements u_0, v_0 and w, and the rotations ϕ_x and ϕ_y of the normal in xz and yz planes, respectively, within the scope of linear shell theory, as

$$\begin{aligned} u(x,y,z,t) &= \left(1+\frac{z}{R_{xx}}\right)u_0(x,y,t) + z\phi_x(x,y,t) \\ v(x,y,z,t) &= \left(1+\frac{z}{R_{yy}}\right)v_0(x,y,t) + z\phi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \quad (3)$$

where t is the time. R_{xx}, R_{yy} in the above equation represent radius of curvatures in x and y directions, respectively.

The strains in terms of mid-plane deformation can be written as

$$\varepsilon = \begin{Bmatrix} \varepsilon_p \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} \quad (4)$$

The mid-plane strain ε_p, bending strains ε_b and shear strains ε_s in Eq. (4) are written as

$$\varepsilon_p = \begin{Bmatrix} u_{0,x} - w/R_{xx} \\ v_{0,x} - w/R_{yy} \\ u_{0,y} + v_{0,x} - 2w/R_{xy} \end{Bmatrix}; \quad \varepsilon_b = \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{Bmatrix};$$

$$\varepsilon_s = \begin{Bmatrix} w_{,x} + \phi_x \\ w_{,y} + \phi_y \end{Bmatrix} \quad (5)$$

The membrane stress resultants $\{N\}$ and the bending stress resultants $\{M\}$ can be related to the membrane strains ε_p and bending strains ε_b through the constitutive relations by

$$\{N\} = \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \mathbf{A}\varepsilon_p + \mathbf{B}\varepsilon_b; \quad \{M\} = \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \mathbf{B}\varepsilon_p + \mathbf{D}\varepsilon_b \quad (6)$$

where the matrices $[A_{ij}]$, $[B_{ij}]$ and $[D_{ij}]$ $(i,j = 1,2,6)$ are the extensional, bending-extensional coupling and bending stiffness coefficients and are defined as $[A_{ij}, B_{ij}, D_{ij}] = \int_{-h/2}^{h/2} [\bar{Q}_{ij}](1,z,z^2)dz$. $\bar{Q}_{ij}$ are the reduced stiffness coefficients [15].

Similarly the transverse shear force $\{\mathbf{Q}\}$ representing the quantities $\{Q_{xz}, Q_{yz}\}$ is related to the transverse shear strains ε_s through the constitutive relations as

$$\{\mathbf{Q}\} = \mathbf{E}\varepsilon_s \quad (7)$$

where

$$E_{ij} = \int_{-h/2}^{h/2} [\bar{Q}_{ij}]\kappa_i\kappa_j dz$$

Here $[E_{ij}]$ $(i,j = 4,5)$ are the transverse shear stiffness coefficients, κ_i is the transverse shear coefficient for non-uniform shear strain distribution through the plate thickness.

The strain energy functional U is given as

$$U(\delta) = (1/2)\int_A \left[\varepsilon_p^T\mathbf{A}\varepsilon_p + \varepsilon_p^T\mathbf{B}\varepsilon_b + \varepsilon_b^T\mathbf{B}\varepsilon_p + \varepsilon_b^T\mathbf{D}\varepsilon_b + \varepsilon_s^T\mathbf{E}\varepsilon_s\right]dA \quad (8)$$

where $\delta\left(=\left\{u_0^1, v_{1^0}^1, w^1, \phi_x^1, \phi_y^1, \ldots, u_0^n, v_{1^0}^n, w^n, \phi_x^n, \phi_y^n\right\}\right)$ is the vector of generalized degrees of freedom associated with the displacement field functions.

The Kinetic energy functional $T(\delta)$ is given as

$$T(\delta) = \frac{1}{2}\int_A \int_{-h/2}^{h/2} \rho^{(k)}\left[\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right]dAdz \quad (9)$$

where $\rho^{(k)}$ represents the material density of kth layer of the hybrid laminate.

For a shallow shell, $(1/R)$ is small and the integral terms involving the curvature are negligibly small compared to the corresponding flat plate terms. Thus, Eq. (9) is rewritten as

$$\begin{aligned} T = \frac{1}{2}{}_A\Big[& I_1\left\{(u_{0,t})^2 + (v_{0,t})^2 + (w_{0,t})^2\right\} + I_2\left\{(\phi_{x,t})(u_{0,t}) + (\phi_{y,t})(v_{0,t})\right\} \\ & + I_3\left\{(\phi_{x,t})^2 + (\phi_{y,t})^2\right\}\Big]dA \end{aligned} \quad (10)$$

where $(I_1, I_2, I_3) = \int_{-h/2}^{h/2}(1,z,z^2)\rho^{(k)}dz$. The average density of a hybrid laminate is given by

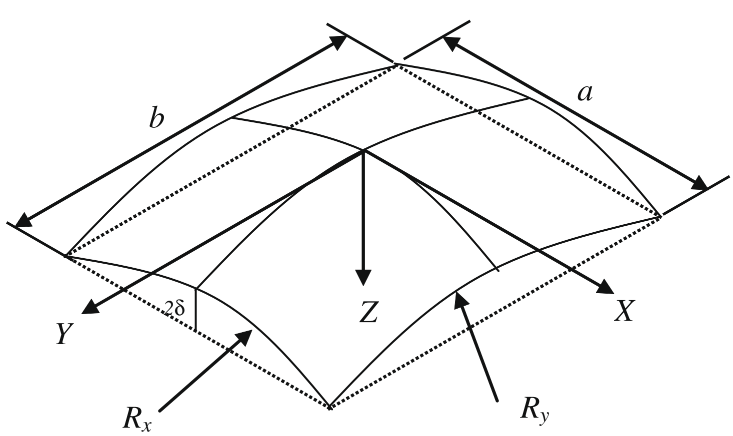

Fig. 1. Curved plate geometry.