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Dynamic stability characteristics of functionally graded materials shallow spherical shells

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Abstract

Here, the dynamic stability behavior of a clamped functionally graded materials spherical shell structural element subjected to external pressure load is studied. The material properties are graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents of the material. The effective material properties are evaluated using a homogenization method. The structural model is based on shear deformation theory and geometric non-linearity is considered in the formulation using von Karman's assumptions. The governing equations obtained are solved employing the Newmark's integration technique coupled with a modified Newton–Raphson iteration scheme. The load corresponding to a sudden jump in the maximum average displacement in the time history of the shell structure is taken as the dynamic buckling pressure. The present model is validated against the available isotropic cases. A detailed numerical study is carried out to bring out the effects of power-law index of functional graded material on the axisymmetric dynamic stability characteristics of shallow spherical shells.

Keywords: Functionally graded; Dynamic buckling; Spherical shell; External pressure load; Power-law index

1. Introduction

The demand for improved structural efficiency in space structures and nuclear reactors has resulted in the development of a new class of materials, called functionally graded materials (FGMs) [1,2]. FGMs are microscopically inhomogeneous, in which the material properties vary smoothly and continuously from one surface of the material to the other surface and thus, distinguish FGMs from conventional composite materials. Typically, these materials are made from a mixture of ceramic and metal, or a combination of different materials. Further, varying the properties in FGMs in a continuous manner is achieved by gradually changing the volume fraction of the constituent materials. The advantages of using these materials are that they are able to withstand high-temperature gradient environments while maintaining their structural integrity, and they avoid the interface problem that exists in homogeneous composites. Furthermore, a mixture of ceramic and metal with a continuously varying volume fraction can be easily manufactured [3–5]. Although these materials are initially designed as thermal barrier materials for aerospace structural applications and fusion reactors, they are now employed for general use as structural elements for different applications [6]. For example, a common structural element for such applications is the rectangular plate, for which several recent studies on statics, buckling, vibration and dynamic behaviors have been performed [7–12].

Applications of functionally graded shell structures are mainly limited to thermal stress, deformation, and fracture analysis in the literature [13–18]. Makino et al. [13], Obata and Noda [14], and Takezono et al. [15] have investigated thermal stress of FGM shells whereas the discs and rotors have been examined based on analytical approach by Durodola and Adlington [16]. The elasto-platics deformations of FGM shell is studied in the work of Dao et al. [17], and Weisenbek et al. [18]. Few transient dynamic analysis of cracked FGM structural components are also reported in the literature [19–21]. Li et al. [19,20] have ana-

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lyzed the stress intensity factor of FGMs under dynamic situation whereas Zhang et al. [21] studied the dynamic responses of cracked FGM structural components. The vibration and parametric instability analysis of functionally graded cylindrical shells under harmonic axial loading have been carried out in Refs. [22,23]. It must be stressed that it is important to be able to predict the dynamic buckling strength of such spherical shells and this type of analysis has received considerable attention in the literature for isotropic [24–30] and anisotropic shell [31–33] structures. However, to the authors' knowledge, work on the dynamic behavior of functionally graded material spherical shells is not commonly available in the literature, and such study is immensely useful to the designers while optimizing FGMs structures.

In the present work, a three-noded shear flexible axisymmetric curved shell element developed based on the field-consistency principle [33,34] is employed to analyze the dynamic behavior of functionally graded structures. The material properties are graded in the thickness direction according to the power-law distribution in terms of volume fractions of the constituents of the material. Geometric non-linearity is assumed in the present study using von Karman's strain–displacement relations. The non-linear governing equations derived are solved employing Newmark's numerical integration method in conjunction with the modified Newton–Raphson iteration scheme. The dynamic buckling pressure is taken as the pressure corresponding to a sudden jump in the maximum average displacement in the time history of the shell structures as shown in Refs. [24,35]. The present formulation is validated considering isotropic case for which solutions are available.

2. Formulation

An axisymmetric functionally graded shell of revolution (radius a, thickness h) made of a mixture of ceramics and metals is considered with the coordinates s, θ and z along the meridional, circumferential and radial/thickness directions, respectively as shown in Fig. 1. The materials in outer ($z = h/2$) and inner ($z = -h/2$) surfaces of the spherical shell are ceramic and metal, respectively. The locally effective material properties are evaluated using homogenization method that is based on the Mori–Tanaka scheme [36–38]. The effective bulk modulus K and shear modulus G of the functionally gradient material evaluated using the Mori–Tanaka estimates are as

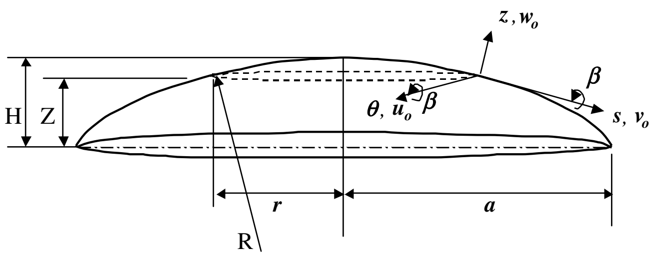

Fig. 1. Geometry and the coordinate system of a spherical cap.

$$\frac{K - K_m}{K_c - K_m} = V_c \Big/ \left[1 + (1 - V_c)\frac{3(K_c - K_m)}{3K_m + 4G_m}\right] \quad (1)$$

$$\frac{G - G_m}{G_c - G_m} = V_c \Big/ \left[1 + (1 - V_c)\frac{(G_c - G_m)}{G_m + f_1}\right] \quad (2)$$

where, $f_1 = \frac{G_m(9K_m + 8G_m)}{6(K_m + 2G_m)}$.

Here, V is volume-fraction of phase material. The subscripts m and c refer the ceramic and metal phases, respectively. The volume-fractions of ceramic and metal phases are related by $V_c + V_m = 1$, and V_c is expressed as

$$V_c(z) = \left(\frac{2z + h}{2h}\right)^k \quad (3)$$

where k is the volume-fraction exponent ($k \geqslant 0$).

The effective values of Young's modulus E and Poisson's ratio υ can be found as from

$$E(z) = \frac{9KG}{3K + G} \quad \text{and} \quad \upsilon(z) = \frac{3K - 2G}{2(3K + G)} \quad (4)$$

The effective mass density ρ can be given by rule of mixture as [39]

$$\rho(z) = \rho_c V_c + \rho_m V_m \quad (5)$$

By using the Mindlin formulation, the displacements at a point (s, θ, z) are expressed as functions of the mid-plane displacements u_0, v_0 and w, and independent rotations β_s and β_θ of the radial and hoop sections, respectively, as

$$\begin{aligned} u(s,\theta,z,t) &= u_0(s,\theta,t) + z\beta_s(s,\theta,t) \\ v(s,\theta,z,t) &= v_0(s,\theta,t) + z\beta_\theta(s,\theta,t) \\ w(s,\theta,z,t) &= w(s,\theta,t) \end{aligned} \quad (6)$$

where t is the time.

Using von Karman's assumption for moderately large deformation, Green's strains can be written in terms of middle-surface deformations as

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p^L \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix} + \begin{Bmatrix} \varepsilon_p^{NL} \\ 0 \end{Bmatrix} \quad (7)$$

where, the membrane strains $\left\{\varepsilon_p^L\right\}$, bending strains $\{\varepsilon_b\}$, shear strains $\{\varepsilon_s\}$ and non-linear in-plane strains $\left\{\varepsilon_p^{NL}\right\}$ in Eq. (7) are written as [40]