Available online at www.sciencedirect.com

ScienceDirect

Composite Structures 79 (2007) 356–368

COMPOSITE STRUCTURES

www.elsevier.com/locate/compstruct

# Dynamic analysis of parametrically excited laminated composite curved panels under non-uniform edge loading with damping

Ratnakar S. Udar, P.K. Datta *

*Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur 721302, India*

Available online 10 March 2006

**Abstract**

The present work deals with the problem of the occurrence of combination resonances in parametrically excited simply supported laminated composite doubly curved panels under non-uniform edge loading. The first-order shear deformation theory is used to model the panels, considering the effects of transverse shear deformation and rotary inertia. Finite element technique is applied for obtaining the non-uniform stress distribution within the curved panels. The method of multiple scale is used to derive the analytical expressions for the instability regions. It is shown that, besides the principal instability region at $\Omega = 2\omega_1$, other cases of $\Omega = \omega_m + \omega_n$, related to other modes, can be of major importance and yield a significantly enlarged instability regions. The effects of non-uniform loading, damping, number of layers, orthotropy, thickness and the static load factor on the dynamic instability behavior of the simply supported laminated composite curved panels are studied. The results show that under localized edge loading, combination resonance zones are as important as simple resonance zones. The effects of damping show that there is finite critical value of dynamic load factor for each instability region below which the curved panels cannot become unstable.


*Keywords:* Combination resonance; The method of multiple scales; Finite element method; Composite curved panels; Non-uniform edge loading; Damping

## 1. Introduction

Fiber-reinforced composite materials are extensively used in laminated thin walled weight sensitive structural parts for various modern engineering structures in the aerospace, mechanical and civil engineering disciplines. In the modern era of technology composite laminates are widely used for aircrafts, space vehicles, under water transportation and many other lightweight applications due to its superiorities like lightweight and high strength. Aircraft and spacecraft structures consist of large number of curved panel type elements, which are subjected to a variety of static and dynamic loads during the flight. Aircraft skin panels are usually subjected to non-uniform in-plane stresses caused by non-uniform loading at the edges. These elements being thin are prone to buckling and dynamic instabilities under external loading. The structural instability may lead to large deflection or large amplitude vibrations of structural elements leading to local or global failures. In recent years, due to the intensive use of fiber-reinforced composites in aeronautic and space structures, the study of the dynamic stability of laminated curved panels has received much attention. Hence, the study of static and dynamic behavior of the laminated composite curved panels subjected to non-uniform compressive edge loading is of considerable importance.

The curved panels subjected to in-plane dynamic (periodic) forces experience resonant transverse vibrations under certain combination of the natural frequency of transverse vibration, the frequency of the in-plane forcing

* Corresponding author. Fax: +91 32 22255303.
*E-mail addresses:* sanjaysu1114@yahoo.co.in (R.S. Udar), pkdatta@aero.iitkgp.ernet.in (P.K. Datta).

0263-8223/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2006.01.013

**Nomenclature**

| | |
|---|---|
| $a$, $b$ | panel lengths in $X$- and $Y$-directions, respectively |
| $c$ | load parameter |
| $[C]$ | damping matrix |
| $E_{11}$, $E_{22}$ | moduli of elasticity |
| $h$ | thickness of the panel |
| $[K]$ | elastic stiffness matrix |
| $[M]$ | mass matrix |
| $P$ | edge load |
| $R_x$, $R_y$ | radii of curvatures in $X$- and $Y$-direction, respectively |
| $[S]$ | stress stiffness matrix |
| $x$, $y$, $z$ | Cartesian co-ordinates |
| $\omega_m$, $\omega_n$ | natural frequencies of the system |
| $\Omega$ | frequency of the excitation |
| $\sigma$ | detuning parameter |
| $\rho$ | mass density of the material |

function and the magnitude of the in-plane load. This phenomenon is called dynamic instability or parametric instability or parametric resonance. The spectrum of the values of parameters causing unstable motion is called the region of dynamic instability or parametric resonance. If the frequency of in-plane forcing function at parametric resonance has relation with only one natural frequency of transverse vibration of the curved panel, the resulting resonance is called simple resonance; otherwise it is called combination resonance. In the analysis of the dynamic stability of a structure subjected to a periodic loading $P(t) = P_s + P_d \cos \Omega t$, it is shown that for certain relationship between the excitation frequency and the natural ones, dynamic instability occurs in the sense that the amplitude of the response increases without bound.

The parametric instability characteristics of laminated composite flat panels subjected to uniform loads were studied by several investigators. Parametric resonance in shell structures under periodic loads has been of considerable interest since the subject was more elaborately introduced by Bolotin [1]. The parametric instability of thick orthotropic cylindrical shells was studied analytically by Bert and Birman [2]. The dynamic instability of laminated composite circular cylindrical shells was studied by Ganapathi and Balamurugan [3] using a $C^0$ shear flexible two-noded axisymmetric shell element. The dynamic stability of cross-ply laminated composite cylindrical shells under combined static and periodic axial force was investigated by Ng et al. [4] using Love's classical theory of thin shells. The parametric instability of laminated composite conical shells under periodic edge loading was studied by Ganapathi et al. [5]. The effects of curvature and aspect ratio on dynamic instability for a uniformly loaded laminated composite thick cylindrical panel were studied by Ganapathi et al. [6]. Recently, Sahu and Datta [7] have studied the parametric resonance characteristics of laminated composite doubly curved shells subjected to non-uniform loading.

The influence of damping on the dynamic behavior of the curved panels becomes more pronounced with the use of special damping treatment to enhance the damping properties of the structure for vibration control. The effect of damping on the dynamic instability behavior of beams has been studied by Engel [8]. Parametric excitation behavior of plates with damping subjected to uniform in-plane loading has been studied by Hutt and Salam [9]. The effect of damping on vibration behavior of plates by the use of viscoelastic layer is investigated by Ioannides and Grootenhuis [10]. Moorthy et al. [11] studied the effect of damping on the parametric instability of laminated composite plates subjected to uniform loading with transverse shear deformation. In all these studies the principal instability region, at which $\Omega = 2\omega_1$ ($\omega_1$ is the lowest natural frequency of the system) was derived and the effects of the aspect ratio, static in-plane force, orthotropy, lamination angles and the number of layers were investigated.

The existence of the combination resonance phenomenon is well established in dynamic instability studies. Bolotin's [1] method is very common for solving simple resonance problems, where the primary instability regions were constructed by using the Fourier analysis, but it cannot be used for solving combination resonance problems. Different authors [12–15] have proposed different methods for solving combination resonance problems. Iwatsubo et al. [16] studied the simple and combination resonances of columns under periodic axial loads. Takahashi and Konishi [17] analyzed the simple as well as combination resonances of rectangular plates subjected to an in-plane sinusoidal linearly varying force by using harmonic balance method. Ostiguy et al. [18] investigated the occurrence of simultaneous and combination resonances in multi-degrees-of-freedom systems subjected to parametric excitation by using the generalized asymptotic method. Saha et al. [19] have studied the simple and combination resonances of a rectangular plate on non-homogeneous Winkler foundation, subjected to uniform compressive in-plane biaxial dynamic loads. Deolasi and Datta [20] have studied the simple and combination resonances of isotropic rectangular plates subjected to non-uniform edge loading with damping. Kar and Sujata [21] investigated the simple and combination resonances of tapered symmetric sandwich beam subjected to periodic axial force by using the method of multiple scales (MMS). Cederbaum [22] studied the simple and combination resonances of shear deformable laminated plates, modeled within HSDT subjected to biaxial periodic loading by using the method of multiple scales (MMS). Mond and Cederbaum [23] investigated