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Free vibration of laminated composite conical shells with random material properties

Vivek Tripathi [a], B.N. Singh [b,*], K.K. Shukla [c]

[a] Department of Aerospace Engineering, Indian Institute of Technology, Kanpur 208 016, India
[b] Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur 721 302, WB, India
[c] Department of Applied Mechanics, Motilal Nehru National Institute of Technology, Allahabad 211 004, UP, India

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Abstract

In the present study, the sensitivity of randomness in material parameters on linear free vibration response of conical shells is presented. Higher order shear deformation theory is used to model system behavior and uncertain lamina material properties are modeled as basic random variables. A finite element method is successfully combined with first-order perturbation technique to obtain the response statistics of the structure. The solution methodology is validated with the results available in the literature and an independent Monte Carlo simulation. Typical numerical results for second-order statistics of linear free vibration response of simply supported laminated composite conical shells are obtained for different lamination schemes and thickness to radius ratios.

Keywords: Random material properties; Conical shell; Composite; Free vibration; Second-order statistics

1. Introduction

The laminated composite conical shells are being widely used in construction of engineering structures is an important field of current area of research. The composite shells made up of composite materials are one of the important structural elements in a variety of high performance engineering systems including aircraft, submarine, and space structures. Composite materials have inherent dispersions in material properties due to lack of complete control over the manufacturing. These materials posses uncertainties in the material properties as compared to their conventional counterparts due to large number of parameters involved in their fabrication. Some important reasons for composite material property variations are air entrapment, delamination, lack of resin, incomplete curing of resin, excess resin between layers and number of geometrical parameters involved such as alignment of fibers, volume fractions, inclusions, voids and others. The variations in the properties of composite materials necessities the inclusion of randomness of material properties in the analysis, otherwise the predicted response may differ significantly rendering the structure unsafe.

For reliability of design, specially for sensitive engineering applications, accurate prediction of system behavior of the composite structures in presence of randomness in system properties favors a probabilistic analysis approach for composites by modeling their mechanical lamina properties as basic random variables (RVs).

Considerable efforts have been made in the past by researchers and investigators on the predictions of the dynamic and buckling response of laminated composite structures considering the material properties as deterministic. Notably among them are due to Moita et al. [1], Correia et al. [2,3], Tighe and Palazotto [4], and Palazotto and Dennis [5]. Extensive literature is available on the response

* Corresponding author.
E-mail addresses: vivekt@iitk.ac.in (V. Tripathi), bnsingh@aero.iitkgp.ernet.in, bnsingh9@rediffmail.com (B.N. Singh), shuklakk@rediffmail.com (K.K. Shukla).

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analysis of the structures with deterministic material properties to random excitations [6]. However, a limited literature is available on analysis of composite structures with uncertain material properties. Ibrahim [7] and Manohar and Ibrahim [8] presented excellent reviews of structural dynamic problems with parameter uncertainties. Leissa and Martin [9] analyzed composite material panels with variable fiber spacing using classical laminate theory (CLT). Employing first-order perturbation technique (FOPT) and CLT, Salim et al. [10–12] analyzed the composite rectangular plates with random material properties. Naveenthraj et al. [13] presented the effect of randomness in material properties and deterministic loading on static response statistics of graphite – epoxy composite laminates, using combination of finite element method (FEM) and Monte Carlo simulation (MCS). Yadav and Verma [14–16] investigated the free vibration of composite circular cylindrical shells with random material properties employing CLT and FOPT. Using FEM in conjunction with FOPT with random material properties, Singh et al. [17–21] investigated free vibration and buckling of laminated composite plates, cylindrical and spherical panels with higher order shear deformation theory (HSDT). Onkar and Yadav [22,23] investigated non-linear response statistics of composite laminates with random material properties under random loading and non-linear free vibration of laminated composite plate with random material properties.

It is evident from the available literature that the studies on the free vibration response of the laminated composite conical shells with random material properties are not dealt by the researchers due to complexity associated with the conical shells. In the present work, an attempt is made to address this problem. The conical shells are analyzed using the FEM. The FOPT is used to handle randomness in material properties. Typical numerical results for dispersion of fundamental frequency of free vibration are obtained for different stacking sequences and thickness to radius ratios for conical shell with all edges simply supported.

2. Formulation

The geometry of the laminated composite conical shell is shown in Fig. 1. Perfect bonding between the layers is assumed. Based on the HSDT model [4,5], the displacement field (u, v, w) at a point in the laminated composite conical shell is expressed as

$$u(s,\theta,z,t) = u_0 + z\beta_s - \left[z^3\frac{4}{3h^2}\left(\sin\varphi\frac{\partial w_0}{\partial s} + \beta_s\right)\right],$$
$$v(s,\theta,z,t) = v_0 + z\left(\frac{\sin\varphi}{R}v_0 + \beta_\theta\right) + \left[z^2\frac{\sin\varphi}{3R}\left(\frac{1}{R}\frac{\partial w_0}{\partial\theta} + \beta_\theta\right) - z^3\frac{4}{3h^2}\left(\frac{1}{R}\frac{\partial w_0}{\partial\theta} + \beta_\theta\right)\right],$$
$$w(s,\theta,t) = w_0, \tag{1}$$

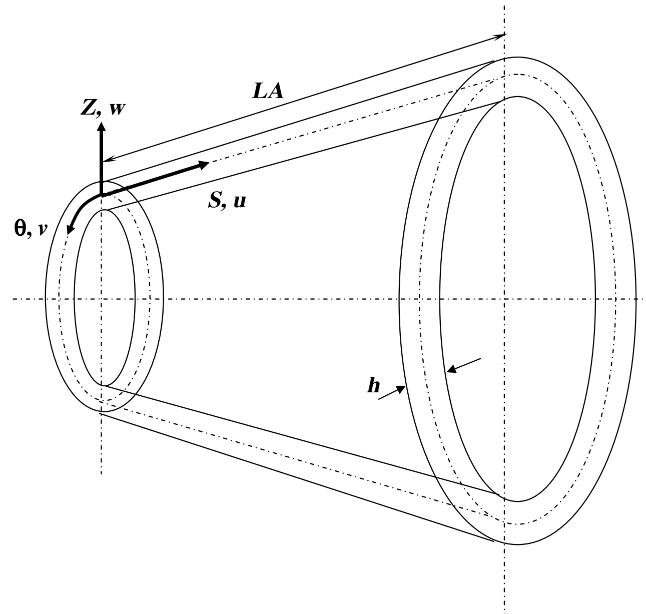

Fig. 1. Shell geometry of conical shell.

where u_0, v_0, w_0 are displacements in the middle plane of the laminate referred to the local axes, β_s, β_θ are the rotations to the normal of the middle plane, about the s and θ-axes and t is the time variable. R is the principal radius of curvature of the surface in the θ-direction. $(90° - \varphi)$ is the half-cone angle [2].

The linear strain–displacement relations for a generic conical geometry are represented by a power series of the transverse coordinate z [2]

$$\begin{aligned}\varepsilon &= \{\varepsilon_{ss} \quad \varepsilon_{\theta\theta} \quad \gamma_{s\theta} \quad \gamma_{\theta z} \quad \gamma_{sz}\}^{\mathrm{T}} \\ &= \{\varepsilon^0_{ss} \quad \varepsilon^0_{\theta\theta} \quad \gamma^0_{s\theta} \quad \gamma^0_{\theta z} \quad \gamma^0_{sz}\}^{\mathrm{T}} \\ &\quad + \sum_{j=1}^{4} z^j\{\kappa^j_{ss} \quad \kappa^j_{\theta\theta} \quad \kappa^j_{s\theta} \quad \kappa^j_{\theta z}\kappa^j_{sz}\}^{\mathrm{T}}.\end{aligned} \tag{2}$$

Considering a shell made of layers of orthotropic material, the fibers of the kth individual layer or ply are oriented at an angle α_k relative to the shell coordinate system. The constitutive relations for kth layer are given by

$$\sigma_k = \overline{Q}^b_k\varepsilon; \quad \tau_k = \overline{Q}^s_k\gamma, \tag{3}$$

where the stress and strain vectors are

$$\begin{aligned}&\varepsilon = \{\varepsilon_{ss} \quad \varepsilon_{\theta\theta} \quad \gamma_{s\theta}\}^{\mathrm{T}}; \quad \gamma = \{\gamma_{\theta z} \quad \gamma_{sz}\}^{\mathrm{T}}, \\ &\sigma_k = \{\sigma_{ss} \quad \sigma_{\theta\theta} \quad \tau_{s\theta}\}^{\mathrm{T}}_k; \quad \tau_k = \{\tau_{\theta z} \quad \tau_{sz}\}^{\mathrm{T}}_k.\end{aligned} \tag{4}$$

The matrices: $\overline{Q}^b_K = \overline{Q}_{ij}(i,j = 1,2,6)$ and $\overline{Q}^s_K = \overline{Q}_{ij}(i,j = 4,5)$ are symmetric arrays of transformed stiffnesses for the kth layer with respect to shell coordinate system (s,θ,z).

The finite element analysis of the conical shell is performed using an eight noded isoparametric serendipity element. The displacement vector over each element is expressed as