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Vibration analysis of functionally graded annular sectorial plates with simply supported radial edges

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Abstract

Assuming that the mechanical properties of the materials vary continuously along the thickness direction and have the same exponential distribution, the free and forced vibration of functionally graded annular sectorial plates with simply supported radial edges and arbitrary circular edges is studied in this paper using a semi-analytical approach (SSM-DQM). The new SSM-DQM method can give an analytical solution along the graded direction using the state space method (SSM) and an effective approximate solution along the radial direction using the one-dimensional differential quadrature method (DQM). The accuracy and convergence of the presented method are demonstrated through numerical examples. For free vibration problems, the influences of various thickness ratios, radii ratios, sector angles, the material property graded indexes and circumferential wave numbers on the lowest non-dimensional frequency are investigated under different circular boundary conditions. And for forced vibration problems, the dynamic response of functionally graded annular sectorial plates under different forcing frequency is studied.

Keywords: Functionally graded materials; Annular sectorial plates; Free vibration; Forced vibration; Semi-analytical method

1. Introduction

Functionally graded materials (FGMs) are composite materials that are microscopically inhomogeneous, and the mechanical properties vary continuously in one (or more) direction(s). This is achieved by gradually changing the composition of the constituent materials along one direction, usually in the thickness direction only, to obtain smooth variation of material properties and optimum response to externally applied loading. The concept of FGMs was first introduced in Japan in 1984. Since then it has gained considerable attentions [1,2]. A lot of different applications of FGMs can be found in references [3].

Dynamical performance of FGM structural components have been extensively studied in the literatures. Yang and Shen [4] investigated the dynamical behavior of initially stressed functionally graded rectangular thin plates subjected to partially distributed impulsive lateral loads and without or resting on an elastic foundation. Vel and Batra [5] presented a three-dimensional exact solution for free and forced vibrations of simply supported functionally graded rectangular plates. Huang and Shen [6] dealt with the nonlinear vibration and dynamical response of functionally graded material plates in thermal environments. Wu et al. [7] obtained solutions to the dynamical equation of inhomogeneous, functionally graded simply supported beams using the semi-inverse method. Chen [8] derived the nonlinear partial differential equations of nonlinear vibration for a functionally graded plate in a general state of non-uniform initial stress and the nonlinear vibration of an initially stressed functionally graded plate was studied. Patel et al. [9] analyzed the free vibration characteristics of functionally graded elliptical cylindrical shells using finite element procedure. Woo et al. [10] provided an analytical solution for the nonlinear free vibration behavior of FGM plates with power-law dependent material properties. Ferreira et al. [11] used the global collocation method

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and approximated the trial solution with multiquadric radial basis functions to analyze free vibrations of functionally graded plates. Shakeri et al. [12] performed the analysis of functionally graded thick hollow cylinders under dynamic load. Zhong and Yu [13] studied the free and forced vibration of a functionally graded piezoelectric rectangular plate by means of a state space approach. Wu et al. [14] investigated the dynamic stability of thick functionally graded material plates subjected to aero-thermo-mechanical loads using the moving least squares differential quadrature method. Chen and Tan [15] studied the nonlinear vibration of initially stressed functionally graded plates with geometric imperfection. Roque et al. [16] performed the free vibration analysis of functionally graded plates by the multiquadric radial basis function method and a higher-order shear deformation theory. Li and Fan [17] analyzed the transient response of a crack embedded in a functionally graded material layer sandwiched between two dissimilar elastic layers under anti-plane shear impact loads. Ganapathi [18] studied the dynamic stability behavior of a clamped functionally graded materials spherical shell structural element subjected to external pressure load.

Many of the above-mentioned papers dealt with the dynamical performance of FGM components having regular geometric shape such as rectangle or circle. However, in many cases, the actual components have special shapes such as sector, annular sector or ellipse. Hence, studies of dynamical characteristics of FGM irregular plates are also important. But, to the authors' knowledge researches on the vibration of FGM annular sectorial plates have not been seen until now. For homogenous annular sectorial plates, there are a few references. McGee et al. [19] obtained an exact solution to the free vibrations of thick (Mindlin) annular sectorial plates having simply supported radial edges and arbitrary conditions along the circular edges. Huang [20] presented an analytical solution for vibrations of a polarly orthotropic Mindlin sectorial plate with simply supported radial edges.

In the present work, the free and forced vibration characteristics of FGM annular sectorial plates with simply supported radial edges and arbitrary circular edges are studied. The material properties are assumed to be graded in the thickness direction according to an exponential distribution. The basic formulations are based on the three-dimensional theory of elasticity. A semi-analytical method, which makes use of the state space method [21,22] along the graded direction and the one-dimensional differential quadrature method [23,24] along the radial direction, is employed. The accuracy and convergence of the present method are demonstrated through numerical results. A detailed parametric study is carried out to highlight the influences of thickness ratios, radii ratios, sector angles, material property graded indexes, circumferential wave numbers, forcing frequencies and circumferential boundary conditions on the vibration frequencies and dynamical responses of FGM annular sectorial plates.

2. Basic equations

Consider an annular sectorial plate having inner radius b, outer radius a, and thickness h which is made of a transversely isotropic functionally graded material, as shown in Fig. 1. A cylindrical polar coordinate system r, θ, z with the origin o on the center of the bottom plane is introduced to describe the displacements of the plate.

The equations of motion in polar coordinates, in the absence of body forces, are

$$\begin{aligned}
&\sigma_{r,r} + \frac{1}{r}\tau_{r\theta,\theta} + \tau_{rz,z} + \frac{1}{r}(\sigma_r - \sigma_\theta) = \rho\ddot{u}_r \\
&\tau_{r\theta,r} + \frac{1}{r}\sigma_{\theta,\theta} + \tau_{\theta z,z} + \frac{2}{r}\tau_{r\theta} = \rho\ddot{u}_\theta \\
&\tau_{rz,r} + \frac{1}{r}\tau_{\theta z,\theta} + \sigma_{z,z} + \frac{1}{r}\tau_{rz} = \rho\ddot{u}_z
\end{aligned} \tag{1}$$

where $\sigma_r, \sigma_\theta, \sigma_z, \tau_{\theta z}, \tau_{rz}, \tau_{r\theta}$ are stress components, u_r, u_θ, u_z are displacement components, ρ is material density and "χ" denotes differentiation with respect to the independent variable χ. The differentiation with respect to time is denoted by a dot.

For a transversely isotropic functionally graded plate, the stress components are related to the displacement components through the following relations:

$$\begin{aligned}
&\sigma_r = c_{11}u_{r,r} + c_{12}\left(\frac{1}{r}u_{\theta,\theta} + \frac{1}{r}u_r\right) + c_{13}u_{z,z} \\
&\tau_{\theta z} = c_{44}\left(\frac{1}{r}u_{z,\theta} + u_{\theta,z}\right) \\
&\sigma_\theta = c_{12}u_{r,r} + c_{11}\left(\frac{1}{r}u_{\theta,\theta} + \frac{1}{r}u_r\right) + c_{13}u_{z,z} \\
&\tau_{rz} = c_{44}(u_{r,z} + u_{z,r}) \\
&\sigma_z = c_{13}u_{r,r} + c_{13}\left(\frac{1}{r}u_{\theta,\theta} + \frac{1}{r}u_r\right) + c_{33}u_{z,z} \\
&\tau_{r\theta} = \frac{c_{11} - c_{12}}{2}\left(u_{\theta,r} + \frac{1}{r}u_{r,\theta} - \frac{1}{r}u_\theta\right)
\end{aligned} \tag{2}$$

where $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ are elastic stiffness components. And we assumed the material properties having the following exponential distributions along the thickness (axial z) of the plates.

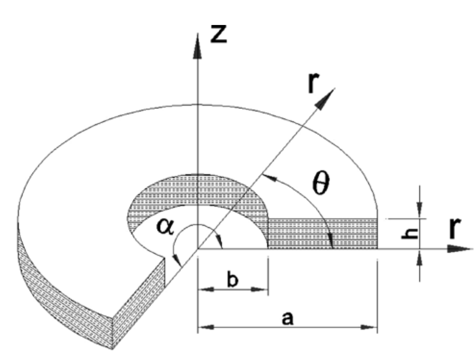

Fig. 1. A FGMs annular sectorial plate with simply supported radial edges.