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Supersonic flutter prediction of functionally graded cylindrical shells

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Abstract

The supersonic flutter analysis of simply supported FG cylindrical shell for different sets of in-plane boundary conditions is performed. The aeroelastic equations of motion are constructed using Love's shell theory and von Karman–Donnell-type of kinematic nonlinearity coupled with linearized first-order potential (piston) theory. The material properties are assumed to be temperature-dependant and graded across the thickness of the shell according to a simple power law. The temperature distribution is assumed to vary in the thickness direction and is obtained by solving the steady-state heat conduction equation. The pre-stresses due to the thermal and mechanical loadings are obtained by exact solution of the equilibrium equations. The Galerkin method is used to solve the aeroelastic equations of motion employing appropriate displacement functions. The effects of internal pressure and temperature rise on the flutter boundaries of the simply supported FG cylinder with different values of power-law index are investigated.

Keywords: Functionally graded; Cylindrical shell; Supersonic; Flutter; In-plane boundary conditions

1. Introduction

Functionally graded materials (FGMs) potentially have wide application in many industries such as aerospace structures. Continuous variation of constituent material's volume fraction is a special property of FGMs that causes the continuous variation of properties. Due to this continuous variation of material properties, these materials can be used in thermal protection systems. Outer skins of most modern aerospace vehicles are made of cylindrical shells and they may experience very high temperature environments. So the use of FGM as their material can be very efficient.

The vibration and buckling of cylindrical functionally graded (FG) shells have been studied by a number of researchers [1–9]. The supersonic flutter analysis of isotropic and composite cylindrical shells with linear shell model have been investigated by many researchers as Olson and Fung [10] Barr and Stearman [11], Carter and Stearman [12], Ganapathi et al. [13] and Pidaparti and Yang [14]. In the work done by Carter and Stearman [12] the flutter boundaries of simply supported cylindrical shell under internal pressure and axial supersonic flow have been obtained for two cases of in-plane BC's: zero traction ($N_{s\theta} = 0$) and classical simply supported boundary conditions. As the knowledge of the authors, there is not any published paper dealing with flutter analysis of FG cylindrical shell subjected to supersonic flow while it's importance in aerospace structures with supersonic and hypersonic speeds. Though, some studies on FG flat plate flutter prediction have been done by Prakash and Ganapathi [15] and Navazi and Haddadpour [16] and post-flutter analysis by Haddadpour et al. [17].

In this paper, the supersonic flutter of simply supported FG cylindrical shells is investigated by solving the linear aeroelastic equations of motion. The material properties are assumed to be temperature-dependant and graded across the thickness according to the simple power low distribution. The Love's shell theory and von Karman–Donnell-type of kinematic nonlinearity is used together with the linearized first-order potential theory to derive the aeroelastic equations of motion. The in-plane inertia terms are neglected in governing equations. The initial stresses

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due to temperature rise or internal pressure is obtained first by exact solution of equilibrium equations and then included in aeroelastic equations of motion. The first two equations of motion (in axial and circumferential directions) are solved with approximate displacement functions and the solution is used with Galerkin method to solve the third equation (in radial direction). All the in-plane BC's are also satisfied with these approximate displacement functions. Assuming a uniform temperature distribution over the surface of the shell, the variation of temperature in the thickness direction is obtained by solving the steady-state heat conduction equation. For different sets of simply supported BC's, the flutter boundaries are obtained using different volume fraction indices and plotted versus internal pressure. Also, the effect of temperature on the flutter boundaries is investigated. The results are compared with the results of flutter analysis of homogeneous cylindrical shells.

2. Theoretical formulation

2.1. Functionally graded materials

An FG cylindrical shell made of a mixture of a ceramic and a metal is considered in the present study. The material properties are assumed to be varied in the thickness direction according to a simple power law as a function of constituent materials volume fraction as

$$V_{\mathrm{m}} = \left(\frac{-z+h/2}{h/2}\right)^{N}, \tag{1}$$

where h is the cylinder wall thickness, N is the volume fraction index, and z is the coordinate in the radial direction with origin at mid-surface (see Fig. 1). Subscript m refers to metal in this paper. By the fact that the summation of the volume fractions is equal to one, the volume fraction of the ceramic can be defined as

$$V_{\mathrm{c}} = 1 - \left(\frac{-z+h/2}{h}\right)^{N}, \tag{2}$$

where subscript c refers to ceramic. The effective material properties of the FG shell, p_{eff} can be obtained as

$$P_{\mathrm{eff}} = P_{\mathrm{m}}V_{\mathrm{m}} + P_{\mathrm{c}}V_{\mathrm{c}}. \tag{3}$$

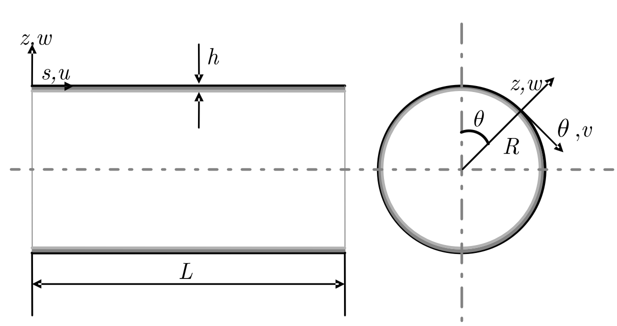

Fig. 1. FG cylindrical shell geometry.

Young's modulus, E, density, ρ, Poisson's ratio, ν, thermal conductivity, K, and thermal expansion coefficient, α, of the FG shell can be determined by use of Eq. (3).

The temperature dependence of material properties can be expressed as [20]

$$P = P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right), \tag{4}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are coefficients of temperature. In the present study, Young's modulus, Poisson's ratio and thermal expansion coefficient are assumed to be temperature-dependent.

The temperature distribution across the thickness of the shell is obtained by solving the steady-state heat conduction equation in the thickness direction

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[K(z)\frac{\mathrm{d}T}{\mathrm{d}z}\right] = 0. \tag{5}$$

The BC's for the above differential equation are determined by the inner surface temperature, T_{m}, which is assumed to be 300 K and the outer surface temperature, T_{c}. By the means of polynomial series solution one can find the solution for Eq. (5) [8] as

$$T(z) = T_{\mathrm{c}} + \frac{(T_{\mathrm{m}} - T_{\mathrm{c}})r\sum_{i=0}^{\infty}\frac{\left(-\frac{r^N K_{\mathrm{mc}}}{K_{\mathrm{m}}}\right)^i}{Ni+1}}{\sum_{i=0}^{\infty}\frac{\left(-\frac{K_{\mathrm{mc}}}{K_{\mathrm{m}}}\right)^i}{Ni+1}}, \tag{6}$$

where

$$K_{\mathrm{mc}} = K_{\mathrm{m}} - K_{\mathrm{c}}, \quad r = \frac{-2z+h}{2h}. \tag{7}$$

2.2. Aerodynamic loading

For a cylinder, which is exposed to an external supersonic flow field parallel to the centerline of the shell, the aerodynamic loading (external pressure) can be obtained by the linearized first-order potential theory with the curvature correction term [11,21]:

$$p_{\mathrm{a}} = -\frac{\gamma_{\mathrm{a}}p_\infty M^2}{(M^2-1)^{1/2}}\left[\frac{\partial w}{\partial s} + \frac{M^2-2}{M^2-1}\frac{1}{U_\infty}\frac{\partial w}{\partial t} - \frac{w}{2R\left(M^2-1\right)^{1/2}}\right], \tag{8}$$

where p_∞, U_∞ and M are free stream static pressure, free stream velocity and Mach number, respectively. Also, w, s and R are the radial deflection, lengthwise coordinate and cylinder radius, respectively (see Fig. 1). For sufficiently high Mach numbers, neglecting the curvature term, i.e. the last term in Eq. (8), yields the linear piston theory

$$p_{\mathrm{a}} = -\gamma p_\infty\left(M\frac{\partial w}{\partial s} + \frac{1}{a_\infty}\frac{\partial w}{\partial t}\right), \tag{9}$$

where a_∞ is the speed of sound. Also γ is the air specific heat ratio and t denotes time. However, Eq. (8) is more accurate for low supersonic speeds and can be used for $M > 1.6$ [22].