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Dynamic analysis of composite cylindrical shells using differential quadrature method (DQM)

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Abstract

Free vibration analysis of composite cylindrical shells with different boundary conditions is presented in this paper using differential quadrature method (DQM). Equations of motion are derived based on first order shear deformation theory taking the effects of shear deformation and rotary inertia terms into account. By applying the differential quadrature formulation and the required modified relationships for implementing the different boundary conditions, equations of motion of a circular cylindrical shell are transformed into a set of algebraic equations. By solving this algebraic system natural frequencies of circular cylindrical shells made of fibrous composite materials with different fibre angles are evaluated. The results thus obtained are then compared with some available results and a good agreement is observed. In all the cases studied here efficiency, ease and usefulness of the DQM are well illustrated.

Keywords: Differential quadrature (DQM); Orthotropic; Shells; Vibration; Frequencies

1. Introduction

High structural performance of mechanical behavior of fibrous composite structures is of a very keen interest in modern engineering. This is due to the extensive use of such structures in various branches of advanced technological and engineering applications. Dynamic analysis of composite cylindrical shells has attracted much attention of researchers e.g. [1–4]. A quick and accurate prediction of dynamic behavior of such cylindrical shells is of very much interest to designers and experimentalists alike. This normally requires a comprehensive development of a mathematical model. Due to the complexity of engineering characteristics of composite shell type structures analytical solutions cannot be obtained in a straight forward manner. The differential quadrature method (DQM) is an efficient numerical technique which transforms governing equations of dynamic equilibrium to a matrix form by using weighted matrices. The DQM requires a small amount of computer capacity and provides accurate results. The DQM was successfully employed in various structural problems [5–8]. In the present study a solution to the free vibration problem of laminated fibrous composite cylindrical shells is presented. The governing equations of dynamic equilibrium are derived based on first order shear deformation theory.

These are then solved by using the differential quadrature method termed as DQM in short. Two types of boundary conditions (simply supported and clamped free ends) for a cylindrical shell are considered in the illustrative examples given in this paper. The results from the present DQM solution are compared with some available theoretical as well as experimental results. It is observed that there is a very good agreement between the results from the present method and the corresponding ones found before by various researchers.

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Nomenclature

a shell radius
C_{ij}^{k} elastic moduli of the kth laminate
A_{ij}, B_{ij}, D_{ij} shell stiffness
E_{ij} DQM weighting coefficients
A_{ij} thickness of the kth layer
K_{xx}, $K_{\theta\theta}$ composite correction factors
L shell length
m axial wave number
n circumferential wave number
n' number of shell layers
M_{xx}, $M_{\theta\theta}$, $M_{x\theta}$ moment resultants
N_{xx}, $N_{\theta\theta}$, $N_{x\theta}$ stress resultants
Q_{xx}, $Q_{\theta\theta}$ transverse shear stress resultants
R_1, R_2, R_3 inertia terms
u, v, w in plane and radial displacements
β_x, β_θ axial and circumferential rotation, respectively
γ_{xz}, $\gamma_{\theta z}$ shear strain components
$u(x)$, $v(x)$, $w(x)$, $\beta_x(x)$, $\beta_\theta(x)$ axial dependence terms in modal forms
ω circular natural frequency
Ω natural frequency parameter $(-a\omega^2)$

2. Theoretical analysis

2.1. Differential quadrature method (DQM)

The differential quadrature method is **a** numerical technique which approximates the spatial derivative of a function at a particular sampling point as a weighted linear sum of the function values at all sampling points chosen in a specified direction. Thus the partial derivatives of a function $f(x_i,\theta_j)$ at a point (x_i,θ_j) are expressed as

$$\left.\frac{\partial^r f(x_i,\theta_j)}{\partial x^r}\right|_{x=x_i} = \sum_{j=1}^{N} E_{ij}^{(r)} f(x_j,\theta_j) \tag{1}$$

where $E_{ij}^{(r)}$ are the respective weighting coefficients and N is the number of grid points and f can be taken as u, v, w, β_x and β_θ. In order to have no constraint on the number of grid points used for the approximation and the weighting coefficients, the Lagrange interpolated polynomials $f_i(x)$ are expressed by

$$f_i(x) = \frac{M(x)}{(x-x_i)M(x_i)_{,x}} \tag{2}$$

where $i = 1,2,\ldots,N$ and $M(x) = \prod_{j=1}^{N}(x-x_i)$ and a comma before a subscript denotes differentiation with respect to that script.

By substituting Eq. (2) in Eq. (1) one can find

$$E_{ij}^{(1)} = \frac{M(x_i)_{,x}}{(x_i-x_j)_{,xx}} \quad (i \neq j) \tag{3}$$

and

$$E_{ij}^{(1)} = \frac{M(x_i)_{,xx}}{2M(x_i)_{,x}} \tag{4}$$

where $i,j = 1,2,\ldots,N$.

A recurrence relation for the rth order weighting coefficients $E_{ij}^{(r)}$ was derived in [8] and is given by

$$E_{ij}^{(r)} = r\left[A_{ij}^{(r-1)}A_{ij}^{(1)} - \frac{A_{ij}^{(r-1)}}{x_i-x_j}\right] \quad (i \neq j) \tag{5}$$

$$E_{ij}^{(r)} = -\sum_{j=1}^{N} A_{ij}^{(r)} \tag{6}$$

where $i,j = 1,2,\ldots,N$.

The grid points are chosen according to Ref. [9].

2.2. Governing equations

Consider a laminated circular cylindrical shell as shown in Fig. 1 for the purpose of mathematical modeling formulation. The formulation is based on Love's thin shell theory equations in terms of circumferential and axial coordinates θ and x, respectively. The governing equations of motion for free vibration and the stress–strain relations including the transverse shear and rotary inertia terms are well known and are taken as proved in [10] and are presented in the following form:

$$\begin{aligned}
&aN_{xx,x} + N_{x\theta,\theta} - Q_{xx} = a\left(R_1\ddot{u} + R_2\ddot{\beta}_x\right)\\
&aN_{x\theta,x} + N_{\theta\theta,\theta} + Q_{\theta\theta} = a\left(R_1\ddot{v} + R_2\ddot{\beta}_\theta\right)\\
&aQ_{xx,x} - Q_{\theta\theta,\theta} - N_{\theta\theta} = a(R_1\ddot{w})\\
&aM_{xx,x} + M_{x\theta,\theta} - aQ_{xx} = a\left(R_2\ddot{u} + R_3\ddot{\beta}_x\right)\\
&aM_{x\theta,x} + M_{\theta\theta,\theta} + aQ_{\theta\theta} = a\left(R_2\ddot{v} + R_3\ddot{\beta}_\theta\right)
\end{aligned} \tag{7}$$

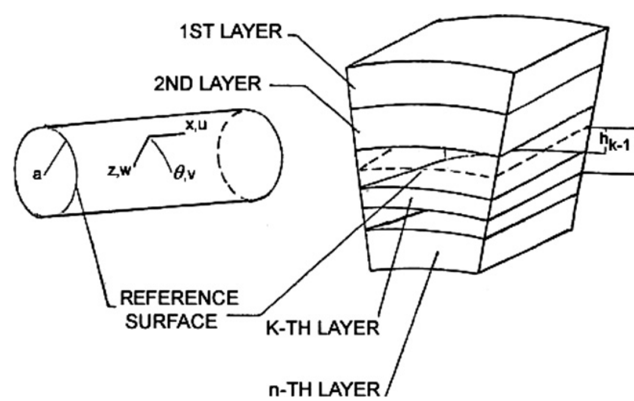

Fig. 1. Cross-sectional view of structure of laminated cylindrical shell.