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The behavior of two parallel interface cracks in magneto–electro–elastic materials under an anti-plane shear stress loading

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Abstract

In this paper, the behavior of two parallel symmetry interface cracks in magneto–electro–elastic materials under an anti-plane shear stress loading is studied by Schmidt method. By using the Fourier transform, the problem can be solved with a pair of dual integral equations in which the unknown variables are the jumps of the displacements across the crack surfaces. To solve the dual integral equations, the jumps of the displacements across the crack surfaces are expanded in a series of Jacobi polynomials. The relations among the electric filed, the magnetic flux and the stress field are obtained. The shielding effect of two parallel interface cracks has been discussed.

Keywords: Magneto–electro–elastic materials; Interface crack; Dual integral equations

1. Introduction

The piezoelectric–piezomagnetic materials are a sort of multi-functionally materials. The piezoelectric–piezomagnetic materials possess piezoelectric, piezomagnetic and magneto-electric effects, thereby making the composite sensitive to elastic, electric and magnetic fields. Consequently, they are extensively used as electric packaging, sensors and actuators, e.g., magnetic field probes, acoustic/ultrasonic devices, hydrophones, and transducers with the responsibility of electro–magneto–mechanical energy conversion [1]. When subjected to mechanical, magnetic and electrical loads in service, these magneto–electro–elastic composites can fail prematurely due to some defects, e.g., cracks, holes, etc. arising during their manufacturing processes. Therefore, it is of great importance to study the magneto–electro–elastic interaction and fracture behaviors of magneto–electro–elastic materials [2–7]. Liu et al. [8] studied the generalized 2D problem of an infinite magneto–electro–elastic plane with an elliptical hole. Gao et al. [9,10] and Wang and Mai [11] also studied the fracture problem of the piezoelectric–piezomagnetic composites. The development of piezoelectric–piezomagnetic composites has its roots in the early work of Van Suchtelen [12] who proposed that the combination of piezoelectric–piezomagnetic phases may exhibit a new material property—the magnetoelectric coupling effect. Since then, there have not been many researchers studying magnetoelectric coupling effect in $BaTiO_3$–$CoFe_2O_4$ composites, and most research results published were obtained in recent years [1–10,13–18]. The static fracture behavior of two parallel symmetry interface cracks in the piezoelectric materials has been investigated in Ref. [19]. However, to our knowledge, the behavior of magneto–electro–elastic materials with two parallel symmetry interface cracks subjected to anti-plane shear stress loading has not been studied by using the Schmidt method [20,21]. Thus, the present work is an attempt to fill this information needed.

In this paper, the behavior of two parallel symmetry interface cracks in magneto–electro–elastic material plane subjected to anti-plane shear loading is investigated by use

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of the Schmidt method [20,21]. The Fourier transform is applied and a mixed boundary value problem is reduced to a pair of dual integral equations. To solve the dual integral equations, the jumps of the displacements across the crack surfaces are expanded in a series of Jacobi polynomials. This process is quite different from those adopted in Refs. [2–11] as mentioned above. The present problem is also quite different from the problem in Ref. [19]. Only the electro–elastic coupling effects were considered in Ref. [19]. Numerical solutions are obtained for the stress. The shielding effect of two parallel interface cracks has been discussed.

2. Formulation of the problem

Fig. 1 shows a piezoelectric–piezomagnetic material layered structure made by bonding together with two same half planes. The piezoelectric–piezomagnetic material layer is layer 2 of thickness h, with two parallel interface cracks of length $2l$ created. A Cartesian coordinate system (x, y) is positioned as shown in Fig. 1. The piezoelectric–piezomagnetic boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement, the in-plane electric fields and the in-plane magnetic fields. As discussed in Soh et al.'s [22] works, since no opening displacement exists for the present anti-plane problem, the crack surfaces can be assumed to be in perfect contact. Accordingly, the electric potential, the magnetic potential, the normal electric displacement and the normal magnetic flux are assumed to be continuous across the crack surfaces. Here, the standard superposition technique is used and only the perturbation fields are considered in the present paper. So the boundary conditions of the present problem are

$$\begin{cases} \tau_{yz}^{(1)}(x,h^+)=\tau_{yz}^{(2)}(x,h^-)=-\tau_0, & |x|\leqslant l \\ w^{(1)}(x,h^+)=w^{(2)}(x,h^-), & |x|>l \\ \tau_{yz}^{(2)}(x,0^+)=\tau_{yz}^{(3)}(x,0^-)=-\tau_0, & |x|\leqslant l \\ w^{(2)}(x,0^+)=w^{(3)}(x,0^-), & |x|>l \end{cases} \tag{1}$$

$$\begin{cases} \phi^{(1)}(x,h^+)=\phi^{(2)}(x,h^-), \ D_y^{(1)}(x,h^+)=D_y^{(2)}(x,h^-), \ |x|<\infty \\ \psi^{(1)}(x,h^+)=\psi^{(2)}(x,h^-), \ B_y^{(1)}(x,h^+)=B_y^{(2)}(x,h^-), \ |x|<\infty \end{cases} \tag{2}$$

$$\begin{cases} \phi^{(2)}(x,0^+)=\phi^{(3)}(x,0^-), \ D_y^{(2)}(x,0^+)=D_y^{(3)}(x,0^-), \ |x|<\infty \\ \psi^{(2)}(x,0^+)=\psi^{(3)}(x,0^-), \ B_y^{(2)}(x,0^+)=B_y^{(3)}(x,0^-), \ |x|<\infty \end{cases} \tag{3}$$

$$w^{(1)}(x,y)=w^{(2)}(x,y)=w^{(3)}(x,y)=0 \quad \text{for } (x^2+y^2)^{1/2}\to\infty \tag{4}$$

where $\tau_{zk}^{(i)}$, $D_k^{(i)}$ and $B_k^{(i)}$ $(k=x,y,\ i=1,2,3)$ are the anti-plane shear stress, in-plane electric displacement and in-plane magnetic flux, respectively. $w^{(i)}$, $\phi^{(i)}$ and $\psi^{(i)}$ are the mechanical displacement, the electric potential and the magnetic potential. Also note that all quantities with superscript i $(i=1,2,3)$ refer to the upper half plane 1, the layer 2 and the lower half plane 3 as shown in Fig. 1, respectively. In this paper, we only consider that τ_0 is positive.

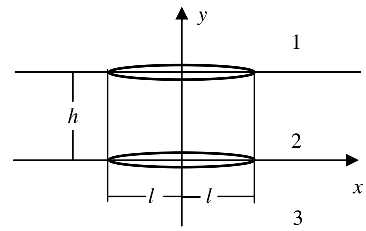

Fig. 1. Two parallel symmetry interface cracks in magneto–electro–elastic materials.

It is assumed that the magneto–electro–elastic material is transversely isotropic. So the constitutive equations for the mode III crack in the magneto–electro–elastic material can be expressed as

$$\tau_{zk}^{(i)}=c_{44}^{(i)}w_{,k}^{(i)}+e_{15}^{(i)}\phi_{,k}^{(i)}+q_{15}^{(i)}\psi_{,k}^{(i)} \quad (k=x,y,\ i=1,2,3) \tag{5}$$

$$D_k^{(i)}=e_{15}^{(i)}w_{,k}^{(i)}-\varepsilon_{11}^{(i)}\phi_{,k}^{(i)}-d_{11}^{(i)}\psi_{,k}^{(i)} \quad (k=x,y,\ i=1,2,3) \tag{6}$$

$$B_k^{(i)}=q_{15}^{(i)}w_{,k}^{(i)}-d_{11}^{(i)}\phi_{,k}^{(i)}-\mu_{11}^{(i)}\psi_{,k}^{(i)} \quad (k=x,y,\ i=1,2,3) \tag{7}$$

where $c_{44}^{(i)}$ is shear modulus, $e_{15}^{(i)}$ is piezoelectric coefficient, $\varepsilon_{11}^{(i)}$ is dielectric parameter, $q_{15}^{(i)}$ is piezomagnetic coefficient, $d_{11}^{(i)}$ is electromagnetic coefficient, $\mu_{11}^{(i)}$ is magnetic permeability, where $c_{44}^{(1)}=c_{44}^{(3)}$, $e_{15}^{(1)}=e_{15}^{(3)}$, $\varepsilon_{11}^{(1)}=\varepsilon_{11}^{(3)}$, $q_{15}^{(1)}=q_{15}^{(3)}$, $d_{11}^{(1)}=d_{11}^{(3)}$ and $\mu_{11}^{(1)}=\mu_{11}^{(3)}$.

The anti-plane governing equations are

$$c_{44}^{(i)}\nabla^2w^{(i)}+e_{15}^{(i)}\nabla^2\phi^{(i)}+q_{15}^{(i)}\nabla^2\psi^{(i)}=0 \quad (i=1,2,3) \tag{8}$$

$$e_{15}^{(i)}\nabla^2w^{(i)}-\varepsilon_{11}^{(i)}\nabla^2\phi^{(i)}-d_{11}^{(i)}\nabla^2\psi^{(i)}=0 \quad (i=1,2,3) \tag{9}$$

$$q_{15}^{(i)}\nabla^2w^{(i)}-d_{11}^{(i)}\nabla^2\phi^{(i)}-\mu_{11}^{(i)}\nabla^2\psi^{(i)}=0 \quad (i=1,2,3) \tag{10}$$

where $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the two-dimensional Laplace operator. Because of the assumed symmetry in geometry and loading, it is sufficient to consider only the problem for $0\leqslant x<\infty$, $-\infty\leqslant y<\infty$. A Fourier transform is applied to Eqs. (8)–(10). It is assumed that the solutions are

$$\begin{cases} w^{(1)}(x,y)=\frac{2}{\pi}\int_0^\infty A_1(s)e^{-sy}\cos(sx)ds \\ \phi^{(1)}(x,y)=\frac{a_1}{a_0}w^{(1)}(x,y)+\frac{2}{\pi}\int_0^\infty B_1(s)e^{-sy}\cos(sx)ds \quad (y\geqslant h) \\ \psi^{(1)}(x,y)=\frac{a_2}{a_0}w^{(1)}(x,y)+\frac{2}{\pi}\int_0^\infty C_1(s)e^{-sy}\cos(sx)ds \end{cases} \tag{11}$$

$$\begin{cases} w^{(2)}(x,y)=\frac{2}{\pi}\int_0^\infty[A_2(s)e^{-sy}+B_2(s)e^{sy}]\cos(sx)ds \\ \phi^{(2)}(x,y)=\frac{a_4}{a_3}w^{(2)}(x,y)+\frac{2}{\pi}\int_0^\infty[C_2(s)e^{-sy} \\ \qquad +D_2(s)e^{sy}]\cos(sx)ds \qquad (0\leqslant y\leqslant h) \\ \psi^{(3)}(x,y)=\frac{a_5}{a_3}w^{(2)}(x,y)+\frac{2}{\pi}\int_0^\infty[E_2(s)e^{-sy} \\ \qquad +F_2(s)e^{sy}]\cos(sx)ds \end{cases} \tag{12}$$

$$\begin{cases} w^{(3)}(x,y)=\frac{2}{\pi}\int_0^\infty A_3(s)e^{sy}\cos(sx)ds \\ \phi^{(3)}(x,y)=\frac{a_1}{a_0}w^{(3)}(x,y)+\frac{2}{\pi}\int_0^\infty B_3(s)e^{sy}\cos(sx)ds \quad (y\leqslant 0) \\ \psi^{(3)}(x,y)=\frac{a_2}{a_0}w^{(3)}(x,y)+\frac{2}{\pi}\int_0^\infty C_3(s)e^{sy}\cos(sx)ds \end{cases} \tag{13}$$