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Effect of elastic coating on fracture behaviour of piezoelectric fibre with a penny-shaped crack

Qing-Hua Qin [a,*], Jian-Shan Wang [b], Xiao-Lei Li [b]

[a] Department of Engineering, Australian National University, FEIT, Canberra, ACT 0200, Australia
[b] Department of Mechanics, School of Mechanical Engineering, Tianjin University, Tianjin 300072, China

Abstract

In this paper, the problem of a penny-shaped crack in a piezoelectric fibre with an elastic coating is investigated. By using the potential function method and Hankel transform, this problem is formulated as the solution of a system of dual integral equations which are reduced to a Fredholm integral equation of the second kind. Numerical studies are conducted to investigate the effect of the thickness and the elastic material properties of the coating on the fracture behavior of piezoelectric fibre composites.

Keywords: Piezoelectric fibre; Penny-shaped crack; Elastic coating

1. Introduction

Piezoelectric composites have been widely used as transducers for sonar projector, underwater and medical ultrasonic imaging applications [1]. Because of the combination of electromechanical properties of piezoelectric fibre and mechanical reliability and flexibility of polymer matrix, many drawbacks of monolithic piezoelectric materials such as low fracture toughness and electric fatigue can be removed by using piezoelectric fibre composites with elastic coating [2]. Therefore, there has been an increasing interest in the study of coated piezoelectric composites.

To prevent the piezoelectric fibre from mechanical failure and increase the bonding strength of the interface between the fibre and matrix during service, the piezoelectric fibre is often coated by an elastic layer [3]. It is desirable to understand effect of the coating layer on fracture behavior of piezoelectric fibre composites. This is the motivation of this paper.

Fracture analysis of piezoelectric materials under electromechanical loading has been of great interest in past decade and a lot of significant efforts have been made to this area [4–7]. Because of mathematical difficulties of the coupled electromechanical fields in piezoelectricity, the majority of the existing works concerned with the two-dimensional crack only such as Griffith crack (see the review papers [8,9]) and the penny-shaped crack in infinite piezoelectric body [10–12]. However, study on penny-shaped crack in piezoelectric composites has received some attention in recent years. Yang and Lee [13] using the potential function approach and Hankel transform and Lin [10] using Fourier and Hankel transform investigated the penny-shaped crack in a piezoelectric fibre. In the current study, a penny-shaped crack in a piezoelectric fibre composite with a finite elastic coating layer under electrical and mechanical loads is investigated to demonstrate the effect of the coating on fracture performance of the piezoelectric composite.

In this paper, we consider the penny-shaped crack in a piezoelectric fibre with a finite elastic coating under in-plane mechanical and electrical loads. The method of solution involves the use of potential function approach and Hankel transform to reduce the crack problem into a system of dual integral equations, which are then further reduced to a Fredholm integral equation of the second kind. Numerical studied are conducted to demonstrate the effect of the elastic coating on fracture behaviour of the piezoelectric fibre composite.

* Corresponding author.
E-mail address: Qinghua.qin@anu.edu.au (Q.-H. Qin).

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2. Formulation of the problem

Consider a piezoelectric fibre with a finite elastic coating and containing a centered penny-shaped crack of radius a under axisymmetric electromechanical loading (Fig. 1). For convenience, a cylindrical coordinate system (r, θ, z) originated at the center of the crack is used with the z-axis along the axis of symmetry of the cylinder. The fibre is assumed to be a transversely isotropic piezoelectric material with the poling direction parallel to the z-axis, and the elastic coating is also the transversely isotropic material. They are subjected to the far-field of a normal strain, $\varepsilon_z = \bar{\varepsilon}(r)$ and a normal electric loading, $E_z = \overline{E}(r)$.

The constitutive equations for piezoelectric materials which are transversely isotropic and poled along the z-axis can be written as [8]:

$$\sigma_\theta = c_{12}\frac{\partial u_r}{\partial r} + c_{11}\frac{u_r}{r} + c_{13}\frac{\partial u_z}{\partial z} + e_{31}\frac{\partial \varphi}{\partial z} \tag{1a}$$

$$\sigma_z = c_{13}\frac{\partial u_r}{\partial r} + c_{13}\frac{u_r}{r} + c_{33}\frac{\partial u_z}{\partial z} + e_{33}\frac{\partial \varphi}{\partial z} \tag{1b}$$

$$\sigma_{rz} = c_{44}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) + e_{15}\frac{\partial \varphi}{\partial r} \tag{1c}$$

$$D_r = e_{15}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) - d_{11}\frac{\partial \varphi}{\partial r} \tag{1d}$$

$$D_z = e_{31}\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) + e_{33}\frac{\partial u_z}{\partial z} - d_{33}\frac{\partial \varphi}{\partial z} \tag{1e}$$

in which φ is the electric potential, σ_{ij} and D_i are components of stress tensor and electric displacement vector, and u_r, u_z are elastic displacements in the r-, z-directions respectively.

In the derivation of the analytic solution, the following potential functions are introduced [13].

$$u_r = \sum_{1}^{3}\frac{\partial \Phi_i}{\partial r}, \quad u_z = \sum_{i=1}^{3} k_{1i}\frac{\partial \Phi}{\partial z}, \quad \varphi = -\sum_{i=1}^{3} k_{2i}\frac{\partial \Phi_i}{\partial z}, \tag{2}$$

where $\Phi_i\,(r,z)(i = 1,2,3)$ are the potential functions, k_{1i} and k_{2i} $(i = 1,2,3)$ are unknown constants in the piezoelectric medium.

The substitution of Eq. (2) into the constitutive Eqs. (1a)–(1e), the field equations and gradient equations, we have the following equations:

$$c_{11}\left(\frac{\partial \Phi_i}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r}\right) + [c_{44} + k_1(c_{13} + c_{44}) + k_2(e_{31} + e_{15})]\frac{\partial \Phi_i}{\partial z^2} = 0 \tag{3a}$$

$$[c_{44}k_1 + c_{13} + c_{44} + e_{15}k_2]\left(\frac{\partial \Phi_i}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r}\right) + [c_{33}k_1 + e_{33}k_2]\frac{\partial \Phi_i}{\partial z^2} = 0 \tag{3b}$$

$$[e_{15}k_1 + e_{31} + e_{15} - d_{11}k_2]\left(\frac{\partial \Phi_i}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r}\right) + [e_{33}k_1 - d_{33}k_2]\frac{\partial \Phi_i}{\partial z^2} = 0 \tag{3c}$$

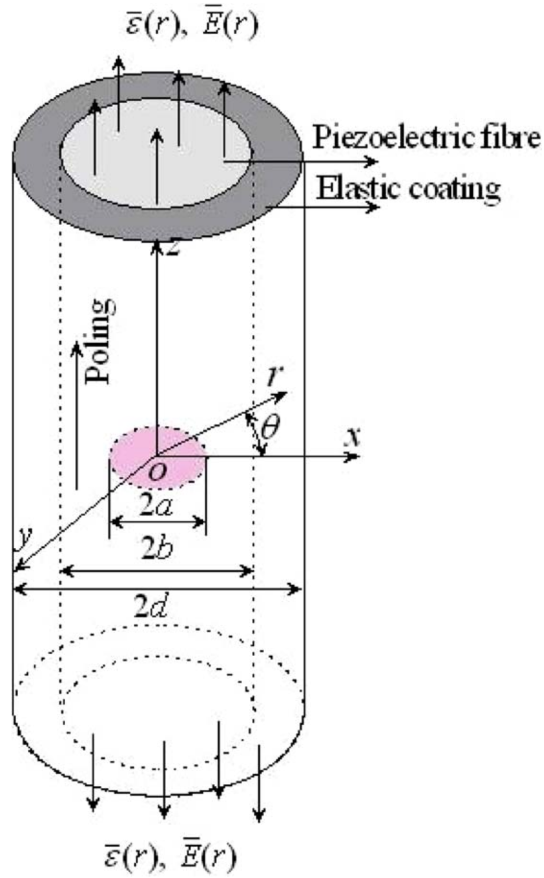

Fig. 1. Piezoelectric fibre with a finite elastic coating and containing a penny-shaped crack under mechanical and electrical loading.

With boundary conditions:

$$\sigma_z(r,0) = 0 \quad (0 \leqslant r < a) \tag{4a}$$

$$u_z(r,0) = 0 \quad (a < r < b) \tag{4b}$$

$$\varphi(r,0) = 0 \quad (a < r < b) \tag{4c}$$

$$\sigma_{rz}(r,0) = 0 \quad (0 \leqslant r < b) \tag{4d}$$

$$D_z(r,0^+) = D_z(r,0^-) \quad (0 \leqslant r < a) \tag{4e}$$

$$E_r(r,0^+) = E_r(r,0^+) \quad (0 \leqslant r < a) \tag{4f}$$

$$D_r(b,z) = 0 \quad (0 < z < \infty) \tag{4g}$$

In this study, following continuity and loading conditions has been used:

(i) The continuity conditions for elastic displacements and tractions at the interface between the fibre and elastic coating are given by:

$$\begin{aligned} &u_z(b,z) = u_z^c(b,z)\ (0 < z < \infty),\\ &u_r(b,z) = u_r^c(b,z)\ (0 < z < \infty),\\ &\sigma_r(b,z) = \sigma_r^c(b,z)\ (0 < z < \infty),\\ &\sigma_{rz}(b,z) = \sigma_{rz}^c(b,z)\ (0 < z < \infty) \end{aligned} \tag{5}$$

(ii) Loading conditions at infinity are:

$$\begin{aligned} &\varepsilon_z(r,\infty) = \bar{\varepsilon}(r), \quad E_z(r,\infty) = \overline{E}(r)\\ &\varepsilon_z^c(r,\infty) = \bar{\varepsilon}(r) \end{aligned} \tag{6}$$