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Vibration and stability of cross-ply laminated composite shallow shells subjected to in-plane stresses

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Abstract

Natural frequencies and buckling stresses of cross-ply laminated composite shallow shells are analyzed by taking into account the effects of transverse shear and normal deformations, and rotatory inertia. By using the method of power series expansion of displacement components, a set of fundamental dynamic equations of a two-dimensional higher-order theory for rectangular laminated shells made of elastic and orthotropic materials is derived through Hamilton's principle. Several sets of truncated approximate theories which can take into account the complete effects of higher-order deformations such as shear deformations with thickness changes and rotatory inertia are applied to solve the vibration and stability problems of laminated composite shallow shells.

Three types of simply supported shallow shells with positive, zero and negative Gaussian curvature are considered. The total number of unknowns does not depend on the number of layers in any multilayered shells. In order to assure the accuracy of the present theory, convergence properties of the lowest natural frequency for the fundamental mode $r = s = 1$ are examined in detail. Numerical results are compared with those of the published three-dimensional models and the extended two-dimensional model in which both in-plane and normal displacements are assumed to be C^0 continuous in the continuity conditions at the interface between layers. It is noticed that the present global higher-order approximate theories can predict accurately the natural frequencies and buckling stresses of simply supported laminated composite shallow shells within small number of unknowns.

Keywords: Cross-ply laminated composite shallow shells; Orthotropic materials; Global higher-order deformation theory; Natural frequency; Buckling stress; Interlaminar stresses

1. Introduction

Structural applications of multilayered composite plates and shells are on the increase due to their stiff, strong and lightweight materials. The mechanical behaviors of laminated composite shells made of high-modulus and low-density materials are strongly dependent on the degree of orthotropy of individual layers, the low ratio of transverse shear modulus to the in-plane modulus and the stacking sequence of laminates. Free vibration and stability problems which have attracted the attention of many researchers up to the present are among the most important problems for laminated composite shells even now. For modelling multilayered composite shells in two-dimensional theories, a review has been made by Noor and Burton [1]. Extensive numerical results for simply supported composite cylinders have been presented and compared with the exact three-dimensional elasticity solutions.

The classical laminated shell theory, based on the Kirchhoff–Love hypothesis, is inaccurate for laminated composite shells with relatively soft transverse shear modulus and for highly anisotropic composites. The inaccuracy is due to neglecting the transverse shear strains and transverse normal strain in the laminate. In order to take into account the effects of low ratio of transverse shear modulus to the in-plane modulus in most of the modern advanced composite materials, a number of first-order shear deformation theories have been developed. However, since in these theories the transverse shear strains are assumed to be constant in the thickness direction, shear correction factors

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have to be incorporated to adjust the transverse shear stiffness for studying the static or dynamic problems of shells. The accuracy of solutions of the first-order shear deformation theory will be strongly dependent on predicting better estimates for the shear correction factors. Since it has been shown that the classical and first-order shear deformation theories are inadequate to predict the accurate solutions of laminated composite shells, the use of more refined two-dimensional multilayered shell theories is required.

In order to obtain the accurate predictions of natural frequencies of simply supported cross-ply cylindrical and doubly-curved laminates, a set of layerwise three-dimensional equations of motion in terms of displacements has been presented by Bhimaraddi [2] and Huang [3]. The three-dimensional layerwise elasticity solution for free vibration of simply supported cylindrical and doubly-curved shallow shells has been presented by Bhimaraddi. Using the assumption that the ratio of thickness to radius of curvature is negligible compared to unity, the governing equations of motion have been reduced to differential equations with constant coefficients. For 2-layer cross-ply laminated shells, the six free surface traction conditions and the six continuity conditions of transverse stresses and displacements have been satisfied at the external surfaces and the interfaces of laminated composite shells, respectively. The free vibration analysis of cross-ply laminated composite shallow cylindrical and doubly-curved panels having an approximately Euclidean surface has been presented by Huang. Natural frequencies have been given for the cross-ply cylindrical, spherical and saddle-shape laminates having various number of layers. The three-dimensional layerwise theory in which both in-plane and normal displacements are assumed to be C^0 continuous in the continuity conditions at the interface between layers. Since the total number of unknowns of three-dimensional layerwise theories depends on the number of layers in a laminate, numerous unknowns for multilayered composite shells are often computationally expensive to obtain the accurate solutions.

Several approximately refined two-dimensional higher-order theories have been proposed to analyze the response characteristics of laminated composite shells. A number of single-layer (global) higher-order shell theories that include the effects of transverse shear deformations have been published in the literature. Although various models of higher-order displacement fields have been considered, Reddy and Liu [4] have presented a higher-order shear deformation theory of laminated composite elastic shell. The theory is the third-order theory in which the in-plane displacements are assumed to be a cubic expression of the thickness coordinate and the out-of-plane displacement to be constant through the thickness. This displacement field leads to the parabolic distribution of the transverse shear stresses and zero transverse normal strain. Bhimaraddi [2] has also presented a two-dimensional higher-order theory for free vibration analysis of doubly-curved shallow shells. The parabolic shear deformation theory in which the transverse shear strains are represented by quadratic functions of the thickness coordinate and the transverse normal strain be zero.

Carrera [5] has applied a mixed variational theorem to vibration analysis of multilayered shells by assuming independently the three displacements and six stress components of the shell. The continuity requirements were explicitly introduced by yielding a formulation depending on the displacement and stress unknowns for each layer with making up a model depending on the number of layers. Based on Reissner's mixed variational theorems, Messina [6] has presented a global theory of simply supported freely vibrating multilayered plates. The global piecewise-smooth functions which can satisfy the external boundary conditions and the interlaminar continuity requirements have been introduced into the two-dimensional plate theory. Messina [7] has generalized the previous investigation [6] for plates to the case of doubly-curved shells. Natural frequencies have been obtained for the cross-ply cylindrical, spherical and saddle-shape laminates having various number of layers and compared with those existing three-dimensional results. By introducing the global piecewise-smooth functions which can satisfy the external boundary conditions and the interlaminar continuity requirements, both in-plane and normal displacements are assumed to be C^0 continuous in the continuity conditions at the interface between layers. Since the total number of unknowns of the present two-dimensional model depends on the maximum number used in the functional expansion and the number of layers in a laminate, numerous unknowns for multilayered composite shells are often computationally expensive to obtain the accurate solutions.

For a thick isotropic shells, a two-dimensional higher-order theory has been developed and has been applied to the free vibration problems of a very thick shallow shell by Matsunaga [8]. Natural frequencies and buckling stresses of thick isotropic shells subjected to in-plane stresses have been analyzed by using the approximate two-dimensional higher-order theories. Remarkable effects of transverse shear deformations and thickness changes have been predicted in the results. A general nonlinear higher-order theory of isotropic shells for large deformations and finite strains in reference to a certain natural state has been presented by Yokoo and Matsunaga [9]. Through the modified Hellinger–Reissner variational principle for a three-dimensional elastic continuum, a set of fundamental shell equations has been derived in terms of the expanded Cauchy–Green strain tensors and Kirchhoff stress resultants. Within the scope of the linear two-dimensional higher-order plate theories, the vibration and stability problems of multilayered composite plates have been investigated by Matsunaga [10]. However, general higher-order theories of shells which take into account the complete effects of shear deformations, thickness changes and rotatory inertia have not been investigated in the vibration and stability problems of multilayered composite shells.

This paper presents a global higher-order theory for analyzing natural frequencies and buckling stresses of