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Composite Structures 74 (2006) 458–466
COMPOSITE STRUCTURES
www.elsevier.com/locate/compstruct

Active damping of piezo-composite beams

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Available online 6 June 2005

Abstract

The aim of the present study is to investigate the effects of piezoceramic materials on the active damping of vibrating piezo-composite beams. The active damping is obtained by using an actuator and a sensor piezoceramic layer acting in 'closed-loop'. By transferring the accumulated voltage on the sensor layer to the piezoelectric actuator layer, the beam can actively damp-out its vibrations. An exact mathematical model, based on a first order shear deformation theory (FSDT) is developed and described. This model allows the investigation of piezo-composite beams with two actuation/sensing type mechanisms, extension and shear. The present study is confined to symmetric lay-up beams using continuous piezoelectric layers. Using the present model, both natural and damped vibrations are calculated. The effects of the feedback gain, *G*, and the beam length on the damped vibrations are investigated and presented.

Keywords: Active damping; Closed-loop; Extension/shear; PZT; Piezo-composite beams

1. Introduction

Many researchers have studied the coupled structure actuator–sensor interactions and have developed both analytical and numerical models to realize the so-called smart or adaptive structures.

The static and free vibration analyses of sandwich beams were carried out to investigate the behavior of sandwich beams with and without piezoelectric materials [3–8].

Knowing the vibration behavior of a composite laminated beam with piezoelectric materials, analytical models have been developed to study the potential of using piezoelectric materials in a closed loop, thus achieving a damping effect [9–18].

Most of the studies presented in the literature concentrate on the extension type piezoelectric mechanism, and only a few works examine the behavior of the shear type piezoelectric mechanism.

Based on the Mindlin plate theory, Sun and Huang [9] presented an analytical model for a cantilever composite beam with piezoelectric (PVDF) sensor and actuator layers. Raja et al. [10] presented an analytical model for a cantilever beam with piezoelectric patches. In that work a comparison was made between the effects of extension-bending and shear PZT on the beam's vibration behavior.

Pitrzakowski [11] presented another analytical model for a Simply–Simply supported beam, with a PZT patch acting as an actuator and a PVDF patch acting as a sensor. Using that model they examined the effect of the piezoelectric layers' stiffness on the active damping efficiency. Chattopadhyay et al. [12] presented a finite element model for calculating the dynamic response of delaminated smart composite cross-ply beams. The model included surface bonded piezoelectric patches acting as actuators and sensors on a cantilever beam. Song et al. [13] presented theoretical and numerical (finite elements analysis) models of laminated composite beams.

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0263-8223/$ - see front matter
doi:10.1016/j.compstruct.2005.04.026

Sloss et al. [18] developed a mathematical model for a cantilever beam with extension piezoelectric patches (PVDF). This model contains a joint contribution of the piezoelectric force and axial force. Using this model, the effect of piezo material on damping the three first frequencies was tested.

The current study is restricted to symmetric lay-ups and uses continuous piezoelectric layers along the beam. We investigated the effect of piezoelectric mechanisms, extension and shear, on the active damping of vibrating piezo-composite beams.

Tow possibilities of using piezoelectric mechanisms as a sensor and actuator in a 'closed-loop' were examined for different common boundary conditions. A parametric investigation was performed to determine the effectiveness of using each of the piezoelectric combinations in terms of the feedback gain (G) values. The investigation was applied for the first three natural frequencies. The present paper provides a brief description of the comprehensive investigation that was performed. The extended description can be found in [1].

In order to complete this investigation, we intend to expand this model for the case of using both piezoelectric mechanisms in a combined 'closed-loop'. Patches of the piezoelectric material will be use. In addition, in order to enhance the validity of the present theoretical research, a series of tests will be performed. The outcome will be reported in due time.

2. Piezoelectric constitutive equations

Fig. 2 depicts a rectangular laminated beam in a Cartesian coordinate system.[1] The structure consists of elastic layers with planes of symmetry coincident with the coordinate system. The extension piezoelectric materials are bonded on the surface of the host structure and poled in the thickness direction, while the shear piezoelectric materials are embedded in the core of the host structure and poled in the plane direction. The application of an electric field in the thickness direction causes the surface actuations to increase or decrease in the plane dimensions. These deformations induce lateral displacements on the host structure; On the other hand, application of electric field, in the thickness direction, will generate transverse deflection of the sandwich structure. As a result of a right rotation of the Cartesian coordinates,[2] we suggested one constitutive relationship for the extension and shear piezoelectric effect,[3] as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & Q_{16} \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 & Q_{26} \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & Q_{36} \\ 0 & 0 & 0 & Q_{44} & Q_{45} & 0 \\ 0 & 0 & 0 & Q_{45} & Q_{55} & 0 \\ Q_{16} & Q_{26} & Q_{36} & 0 & 0 & Q_{66} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} e_{11} & 0 & e_{31} \\ e_{12} & 0 & e_{32} \\ e_{13} & 0 & e_{33} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & e_{35} \\ 0 & e_{26} & 0 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0 \\ E_3 \end{Bmatrix} \tag{1}$$

where σ_{ij} and ϵ_{ij} are the Cauchy tensor for stresses and strains components respectively, E_j is the electric field, Q_{ij} are the elasticity constants and e_{ij} are the piezoelectric module.

Using extension piezoelectric actuators the only nonzero components of the piezoelectric tensor are e_{31}, e_{32}, e_{33}, e_{24}, and e_{15}, while for shear piezoelectric actuation the only nonzero components of the piezoelectric tensor are e_{11}, e_{12}, e_{13}, e_{26}, and e_{35}. Note that due to symmetry, $e_{15} = e_{35}$ and $e_{13} = e_{31}$.

For a beam problem, one can use $\sigma_y = \tau_{xy} = \tau_{yz}$ while assuming that the strains $\varepsilon_y \neq \gamma_{yz} \neq \gamma_{xy} \neq 0$, to obtain the following reduced constitutive equations:

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \widetilde{Q}_{11} & 0 \\ 0 & \widetilde{Q}_{55} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} + \begin{Bmatrix} \widetilde{P}_1 \\ \widetilde{P}_4 \end{Bmatrix} \cdot \{E_z\} \tag{2}$$

where the relations for $\widetilde{Q}_{ij}$ and $\widetilde{P}_i$ in terms of Q_{ij} and e_{ij} respectively, are given by

$$\begin{aligned} &\widetilde{Q}_{11} = Q_{11} - \frac{Q_{13} \cdot Q_{13}}{Q_{33}}, \quad \widetilde{Q}_{55} = Q_{55}, \\ &\widetilde{P}_1 = \frac{Q_{13} \cdot e_{33}}{Q_{33}} - e_{31}, \quad \widetilde{P}_4 = e_{35} \end{aligned} \tag{3}$$

Integrating the stress components across each lamina, and expressing the displacement field based on the first order shear deformation theory (FSDT), one can obtain the stress resultant N_x, the stress couple M_x, and the transverse shear resultant Q_x, per unit width for the overall structure:

$$\begin{Bmatrix} N_x \\ M_x \\ Q_{xz} \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} & 0 \\ B_{11} & D_{11} & 0 \\ 0 & 0 & A_{55} \end{bmatrix} \begin{Bmatrix} U_{,x} \\ \Phi_{,x} \\ W_{,x} + \Phi \end{Bmatrix} + \begin{Bmatrix} E_{11} \\ F_{11} \\ G_{55} \end{Bmatrix} \tag{4}$$

[1] For the derivations presented here it is assumed that the principal materials coordinates coincide with the coordinates of the problem being analyzed.

[2] Rotating by 90° around the out of plane direction, and then imposing a 180° rotation around the transverse direction, we get an interchange between axial (x, or 1) and transverse (w, or 3) indices.

[3] In the traditional form, two separate constitutive relationships are used, one for extension type and other for shear type.