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Analysis of the nonlinear dynamic response of viscoelastic symmetric cross-ply laminated plates with transverse matrix crack

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Abstract

Based on the Schapery's 3-D constitutive relationship and the Von Karman's plate theory, the constitutive equations and the nonlinear dynamic governing equations for viscoelastic symmetric cross-ply laminated plates with transverse matrix cracks are derived. By using the finite difference method and Newmark-β scheme, the unknown functions are separated and these equations are iterated to seek solutions. Numerical calculating results show that the effect of damage on the nonlinear dynamic responses of structures is significant.

Keywords: Damage; Viscoelastic; Laminated plates; Nonlinear dynamic response

1. Introduction

The nonlinear dynamic behavior of laminated elastic plates has been received considerable attention in recent years. Most of the existing solutions, however, did not consider the effect of the material's viscoelastic properties and the damage evolution. Generally, the materials of the composite laminated plates have the property of viscoelasticity with the apparent creep phenomenon and relaxation characteristic. Moreover, various damages will emerge in the laminated structure under the action of loading, temperature and environment. The emergence and development of the damage will decrease the stiffness of the structure and lead to the change of the dynamic behaviors. Huang [1] studied the creep-displacement response of the cross-ply and angle-ply laminated plates with geometric nonlinearity and initial deflection. By using Von Karman equations and Boltzmann superposition principle, Cheng and Zheng [2] revealed the nonlinear dynamic behaviors of the viscoelastic rectangular plate subjected to the lateral periodic load. Librescu and Chandiramani [3] analyzed the dynamic stability of transversely isotropic viscoelastic plates. Sun and Zheng [4] investigated the chaotic behaviors of viscoelastic rectangular plate subjected to an in-plane periodic load and pointed out that the stability of the structure could be increased by adjusting the material parameters. Viggo [5] discussed the creep buckling of the rectangular plates under axial compression. Through applying the finite element method and multiple scales analysis, Kim and Kim [6] investigated the nonlinear dynamic behavior of viscoelastic composite laminated plates. Based on the continuum mechanics, the constitutive equations and the corresponding damage evolution equations were formulated by Ladeveze and Dantec [7] and Allix and Landeveze [8], and in their work matrix cracking, fiber/matrix debonding and interlaminar delamination were all considered. By using Kachanov's continuum damage model, the creep life of creep-brittle solid and structure was analyzed by Rodin [9]. Weisman [10] and Tawab and Weitsman [11] founded a continuum damage model for viscoelastic materials with growing damage, and Akshantala and

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Brinson [12] put forward a damage evolution model for viscoelastic composite laminates. Fan and Shen [13] obtained a damage evolution equation for viscoelastic orthotropic plate through experiments. Zhang and Jin [14] investigated the dynamic response of isotropic elastic beams and plates with anisotropic damage. By adopting the elastic-viscoelastic correspondence principle, Schapery [15] established the three-dimensional constitutive relationship of linear viscoelastic and unidirectional fiber reinforced composite with transverse matrix cracks. Nevertheless, there was little research on the nonlinear dynamic response for viscoelastic composite laminated plates and shells with damage.

In the present study, based on the Schapery's 3-D constitutive relationship, the nonlinear dynamic governing equations of viscoelastic symmetric cross-ply laminated plates with transverse cracks in matrix are derived. By applying the finite difference method and Newmark-β scheme, the unknown functions are separated and these equations are iterated to seek solutions. The effects of different parameters on the nonlinear dynamic response of damaged structure are discussed. Present results are compared with available data.

2. Basic equations

Consider a viscoelastic rectangular plate having length a in the x direction, width b in the y direction, thickness h in the z direction, and mass density ρ of unit area. The midsurface of the plate containing the x, y axis and the coordinate system are shown in Fig. 1. Denote z_k as the distance from the midsurface of the kth layer to the coordinate plane. According to the classical Von Karman's plate theory, at arbitrary time t, the displacement components u_1, u_2 and u_3 may be described by the following expressions:

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) - zw_{,x}(x,y,t) \\ u_2(x,y,z,t) &= v(x,y,t) - zw_{,y}(x,y,t) \\ u_3(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{1}$$

where u, v and w are the values of u_1, u_2 and u_3 at the midsurface, and a comma denotes the partial derivative with respect to the corresponding coordinate. Also, the nonlinear strain–displacement relations can be written as

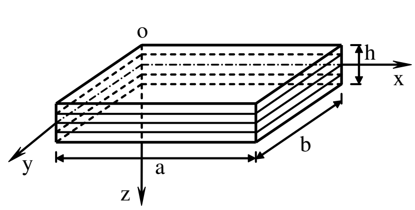

Fig. 1. Viscoelastic composite laminated plate.

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x, \quad \varepsilon_y = \varepsilon_y^0 + z\kappa_y, \quad \gamma_{xy} = \gamma_{xy}^0 + z\kappa_{xy} \tag{2}$$

where $\kappa_x = -w_{,xx}$, $\kappa_y = -w_{,yy}$, $\kappa_{xy} = -2w_{,xy}$, and

$$\begin{aligned} &\varepsilon_x^0 = u_{,x} + \frac{1}{2}w_{,x}^2, \quad \varepsilon_y^0 = v_{,y} + \frac{1}{2}w_{,y}^2, \\ &\gamma_{xy}^0 = u_{,y} + v_{,x} + w_{,x}w_{,y} \end{aligned} \tag{3}$$

On the basis of micromechanics, the unidirectional fiber composite can be assumed to be transversely isotropic, and the stiffness of elastic fibers is much larger than those of isotropic viscoelastic polymeric matrix. As for the laminated plates with transverse cracks which are parallel to the fibers and normal to the ply-plane, they cannot be considered as transversely isotropic, but as a very good approximation, they can be considered as special orthotropic, as indicated by Noh and Whitcomb [16]. In every layer, the Young's modulus of the fibers orientation is denoted by E_{11} and the modulus which perpendicular to the fibers is denoted by E_{22}, while ν_{12} and ν_{23} are the Poisson ratio, G_{12} and G_{23} are shear modulus. But, only E_{22}, G_{12} and G_{23} are affected by the micro-cracks that was illustrated by Noh and Whitcomb for linearly elastic composites, and Schapery also proved that the conclusion is all right for linearly viscoelastic composite materials. Take D_2, D_4 and D_6 as damage variables, and they are defined as follows:

$$\begin{aligned} &D_2 = 1 - \overline{E}_{22}/E_{22}, \quad D_4 = 1 - \overline{G}_{23}/G_{23}, \\ &D_6 = 1 - \overline{G}_{12}/G_{12} \end{aligned} \tag{4}$$

where $\overline{E}_{22}$, $\overline{G}_{12}$ and $\overline{G}_{23}$ are the modulus after damage. Using equivalent effective strain principle, the three-dimensional constitutive relationship for viscoelastic plates with transverse cracks in matrix may be given by the following expressions [15]:

$$\begin{aligned} \sigma_1 &= E_{11}\varepsilon_{11} + \frac{\nu_{12}}{1/(1-D_2) - \nu_{23}^2} \\ &\quad \times \left[(1+\nu_{23})E_{22}\otimes\bar{\varepsilon}_2 + \left(\frac{1}{1-D_2} + \nu_{23}\right)E_{22}\otimes\bar{\varepsilon}_3\right] \\ \sigma_2 &= \frac{1}{1/(1-D_2) - \nu_{23}^2}E_{22}\otimes(\bar{\varepsilon}_2 + \nu_{23}\bar{\varepsilon}_3) \\ \sigma_3 &= \frac{1}{1/(1-D_2) - \nu_{23}^2}\left(\frac{1}{1-D_2}E_{22}\otimes\bar{\varepsilon}_3 + \nu_{23}E_{22}\otimes\bar{\varepsilon}_2\right) \\ \sigma_4 &= \frac{1-D_4}{2(1+\nu_{23})}E_{22}\otimes\varepsilon_4 \\ \sigma_5 &= G_{12}\otimes\varepsilon_5, \quad \sigma_6 = (1-D_6)G_{12}\otimes\varepsilon_6 \end{aligned} \tag{5}$$

in which $\bar{\varepsilon}_2 = \varepsilon_2 + \nu_{12}\varepsilon_1, \bar{\varepsilon}_3 = \varepsilon_3 + \nu_{12}\varepsilon_1$, symbol $\otimes$ is Boltzmann operator and defined as

$$f(t)\otimes g(t) = f(0)g(t) + \int_0^t \dot{f}(t-\tau)g(\tau)\,\mathrm{d}\tau \tag{6}$$

Suppose every layer of the laminated plate is in plane stress state and the fibers are parallel to the direction