Computers and Geotechnics 74 (2016) 15–25

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

A complete formulation of an indirect boundary element method for poroelastic rocks

Abolfazl Abdollahipour *, Mohammad Fatehi Marji, Alireza Yarahmadi Bafghi, Javad Gholamnejad

Faculty of Mining and Metallurgical Engineering, Yazd University, Yazd, P.O. Box 89195-741, Iran

ARTICLE INFO

Article history:
Received 22 August 2015
Received in revised form 24 November 2015
Accepted 19 December 2015
Available online 6 January 2016

Keywords:
Fundamental solutions
Influence functions
Poroelastic rock
Displacement discontinuity method

ABSTRACT

Rocks are naturally filled with cracks and pores that are saturated with one or more fluid phases. Many problems in rock mechanics, petroleum engineering, geophysics, etc. deal with cracks and discontinuities in rock formations. These problems should consider effects of a porous medium. Displacement discontinuity method (*DDM*) as an indirect boundary element method is particularly ideal for problems involving fractures and discontinuities. However, the *DDM* in its original form is limited to elastic problems. The paper uses a fundamental solution of a point displacement discontinuity in poroelastic medium to obtain the solution for a poroelastic *DDM*. Then it introduces a numerical formulation and implementation for the poroelastic *DDM* in a code named *CEP-DDM* (Constant Element Poroelastic *DDM*). The accuracy and validity of the proposed solution and the newly developed code are verified by two analytical solutions, another numerical solution, and some field measurements. These results showed good agreement between *CEP-DDM* and other methods' results. The verifications prove the accuracy and applicability of the proposed numerical model in a wide range of real-world problems.

1. Introduction

Generally, subsurface rocks include discontinuities (such as cracks and faults) and pores where fluids of any phase may exist. These discontinuities and pores may be saturated with water, air, oil, etc. These fluids can significantly affect the stress (e.g. causing effective stresses due to pore pressure effect) and displacement fields in a rock mass. Pore pressures can cause elastic deformation in the rock which results in the flow of pore fluid through the rock due to the pore pressure gradients, changes in stresses due to tectonic forces, boreholes drilling, etc. [1].This results in a strong couple of the mechanical and hydrological behavior of rocks. The coupled hydromechanical rocks behavior should be studied in poroelasticity framework.

The problem of a pressurized fracture in a porous medium arises in many situations in geomechanics such as hydraulic fracturing [2–6], In-situ stress measurement [7–9], and geothermal energy extraction process [10–13]. Fractures are the main flow channels in a poroelastic medium. During the last decades, many papers have focused on the derivation of a mathematical formulation or analytical solution for the hydraulic fracture problem in a porous rock [14–20].

Change in the fluid pressure induces matrix deformation and stress change, matrix deformation in turn induces fluid volume change and fluid pressure change. The change in pore pressure and stress at any point affects the fracture and induces fracture deformation. In order to accurately model these coupled interactions, all these couplings should be implicitly considered. The displacement discontinuity method (*DDM*) is particularly ideal for problems involving fractures and discontinuities because the fundamental solution contains a displacement jump. Although the *DDM* in its original form [21] and its higher order extensions [22–24] gives very accurate results in the solution of boundary value problems (BVPs) but is limited to elastic problems. Therefore, the *DDM* has been coupled with other numerical methods such as *FDM* and *FEM* to investigate poroelastic effects of fractures. For example, Ji used the *DDM* to simulate crack propagation in a poroelastic environment and coupled it with the *FDM* to simulate fluid interaction [25]. Yin et al. coupled *DDM* and *FEM* to consider poroelastic effects in reservoirs [26,27].

However, the *DDM* has not been completely formulated and extended to poroelasticity as a stand-alone numerical method. The present paper derives the fundamental solutions for poroelastic *DDM* in an implicit form (from a mathematical point of view). Then introduces numerical formulation and implementation of the *DDM* in a poroelastic rock. A sophisticated computer code named *CEP-DDM* (constant element poroelastic displacement dis-

* Corresponding author. Tel.: +98 912 45 70 897.
E-mail address: ab.abdollahipour@gmail.com (A. Abdollahipour).

http://dx.doi.org/10.1016/j.compgeo.2015.12.011
0266-352X/

Nomenclature

F_i	bulk body force (N)
g_i	gravity component in i direction (m/s^2)
ζ	variation of fluid content (kg/m^3)
ρ	bulk density (kg/m^3)
ρ_s	fluid density (kg/m^3)
ρ_f	solid density (kg/m^3)
n	porosity
q_i	specific discharge
f_i	fluid body force (N)
γ	fluid injection rate (m^3/s)
α	Biot coefficient of effective stress
σ_{ij}	total stress (Pa)
p	pore pressure (Pa)
e_{ij}	solid strain
G	shear modulus (Pa)
ν_u	undrained Poisson ratio
ν	drained Poisson ratio
K	drained bulk modulus (Pa)
K_u	undrained bulk modulus (Pa)
B	Skempton's pore pressure coefficient
κ	permeability coefficient
k	intrinsic permeability (m^2)
μ	fluid dynamic viscosity (m^2/s)
c	generalized consolidation coefficient (m^2/s)
$(u_{ij})^0$	time-independent displacement (m)
$(\sigma_{ijk})^0$	time-independent stress (Pa)
$(p_i)^0$	time-independent pore pressure (Pa)
$(q_{ij})^0$	time-independent flux (m^3/s)
Δu_{ij}	time-dependent displacement (m)
$\Delta\sigma_{ijk}$	time-dependent stress (Pa)
Δp_i	time-dependent pore pressure (Pa)
Δq_i	time-dependent flux
D_s	shear displacement discontinuity
D_n	normal displacement discontinuity
D_f	flux discontinuity
δ	dirac delta
i, j, k, l	indices varying from 1 to 2
$\bar{x}, \bar{y}, n, s$	local coordinates
m	number of elements
h	number of time steps
Δt	time-step
$A^{ij}_{ns}, \ldots$	boundary influence coefficients
ω	number of current time step
β	angle between two different local axes

continuity method) is developed to solve the general poroelastic boundary value problems. The code is well verified with the known analytical solution of the problem of suddenly pressurized crack in an infinite plane cited in the literature [21,22] as well as a problem with finite boundaries [28]. Also, the results are compared with another numerical method available in the literature[29]. The newly developed formulation and code provide a complete poroelastic analysis without the need to couple the *DDM* with any other numerical method and dealing with the problems involving translating data from one method to another.

2. Poroelasticity

The linear, isotropic poroelasticity theory was pioneered by Biot [30] for modeling fluid-saturated solids response and was further extended by others [31–34]. Poroelasticity considers solid and fluid parts for geomaterials. The original formulation of Biot is considered in this research. This formulation mainly consists of basic dynamic parameters of total stress σ_{ij} and pore pressure p along with their corresponding quantities, solid strain $e_{ij} = (u_{i,j} + u_{j,i})/2$ and variation of fluid volume per unit reference ζ. A consistent set of parameters for linear isotropic theory are shear modulus G, drained and undrained Poisson ratios which are respectively, $\nu = (3K - 2G)/2(3K + G)$, $\nu_u = (3K_u - 2G)/2(3K_u + G)$, drained and undrained bulk moduli K and K_u, Skempton's pore pressure coefficient B (ratio of induced pore pressure to variation of confined pressure in undrained conditions) and permeability coefficient $\kappa = k/\mu$ [30]. Governing equations of linear isotropic poroelasticity consists of the following:

- Constitutive equations:

$$\sigma_{ij} = 2Ge_{ij} + \frac{2G\nu}{1-2\nu}\delta_{ij}e - \alpha\delta_{ij}p \tag{1}$$

$$p = -\frac{2GB(1+\nu_u)}{3(1-2\nu_u)}e + \frac{2GB^2(1-2\nu)(1+\nu_u)^2}{9(\nu_u-\nu)(1-2\nu_u)}\zeta \tag{2}$$

- Static local stress equilibrium equation

$$\sigma_{ij,j} = -F_i \tag{3}$$

- Darcy's law

$$q_i = -\kappa(p_{,i} - f_i) \tag{4}$$

Darcy's law controls fluid flow in poroelastic rocks. Based on Darcy's law, flux field q_i is irrotational due to the fact that it is derived from the gradient of a continuous field. Considering the law of conservation of mass for a compressible fluid the following equation for fluid continuity may be obtained.

$$\frac{\partial\zeta}{\partial t} + q_{i,i} = \gamma \tag{5}$$

where in the above equations, $F_i = \rho g_i$, $\rho = (1-n)\rho_s + \varphi\rho_f$, $f_i = \rho_f g_i$ and α is Biot coefficient of effective stress which is:

$$\alpha = \frac{3(\nu_u - \nu)}{B(1-2\nu)(1+\nu_u)} \tag{6}$$

The foregoing equations can be combined to obtain a set of field equations in terms of displacement and fluid content variation. Combining Eqs. (1)–(3) yields an elasticity equation with a fluid coupling term

$$G\nabla^2 u_i + \frac{G}{1-2\nu_u}\, e_{,i} - \frac{2GB(1+\nu_u)}{3(1-2\nu_u)}\zeta_{,i} = -F_i \tag{7}$$

Combining Eqs. (2), (4) and (5) and using Eq. (7) gives the following diffusion equation.

$$\frac{\partial\zeta}{\partial t} - c\nabla^2\zeta = \frac{kB(1+\nu_u)}{3(1-\nu_u)}F_{i,i} - kf_{i,i} + \gamma \tag{8}$$

where c is a generalized consolidation coefficient equal to [32]

$$c = \frac{2kB^2G(1-\nu)(1+\nu_u)^2}{9(1-\nu_u)(\nu_u-\nu)} \tag{9}$$