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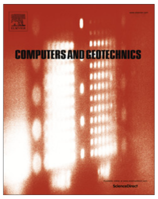

Research Paper

Analytical solution to axisymmetric consolidation in unsaturated soils with linearly depth-dependent initial conditions

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ABSTRACT

This paper introduces an analytical solution for the axisymmetric consolidation of unsaturated soils subjected to constant external loading. The analytical procedure employs variables separation and Laplace transformation techniques while capturing the uniform and linear initial excess pore pressure distributions with depth. Excess pore-air and pore-water pressures as functions of time, radial and vertical flows are determined using Laplace transforms, Fourier Bessel and sine series, respectively. In this study, the consolidation behavior, in terms of changes in excess pore-air and pore-water pressures and the average degree of consolidation, are investigated against the air to water permeability ratio. The effects of radial distance from the drain well on the dissipation rate are likewise highlighted in worked examples. Excess pore pressure isochrones and the matric suction varying with time are also presented.

1. Introduction

In major ground improvement projects, vertical drain assisted preloading has been a cost-effective method to accelerate drainage in soil deposits and shorten the compression process. Vertical drain consolidation can be modeled assuming axisymmetry around the drain. The basic theory of the axisymmetric consolidation was developed based on the traditional theory proposed by Terzaghi [1]. Barron [2] first introduced solutions for axisymmetric consolidation of saturated soils while considering radial flow only. Yoshikuni and Nakanodo [3] introduced a rigorous free strain equation adopting permeable top and bottom boundaries of a cylindrical soil mass while considering the well resistance. Based on the equal strain condition, Hansbo [4] developed a simplified closed-form solution for the degree of consolidation induced by drain wells capturing the well resistance and peripheral smear effects. Further studies have included the effects of the smear zone and time-dependent loading [5–11], varying radial permeability coefficients [12–14], new analysis for soil–drain system conceptually based on the double porosity model (DPM) [15], vertical drain consolidation with discharge capacity varying linearly with depth and decreasing exponentially with time [16], or axisymmetric model including both radial and vertical flows for electro-osmotic consolidation [17].

The above mentioned studies are only valid for saturated soils. In real practice, drain wells may be installed through multi-layered geomaterials, where groundwater table is found to be reasonably below the ground surface, and often unsaturated soil layers may be found near the soil surface. In other words, the capillary zones do not extend to the soil surface in many situations. This unsaturated soil portion could be a result of past earthworks (e.g. excavation, compaction) or climate changes (e.g. arid or semi-arid climates) [18]. It is believed that the presence of unsaturated soil deposits within the entire strata, where the drain wells are installed, may significantly influence the consolidation behavior and likewise characterize the settlement pattern of a soil. Thus, there has been an essential need for a generalized model that can be applicable for unsaturated soils.

Consolidation studies for unsaturated soils have progressed rigorously since the inception of one-dimensional (1D) consolidation theory proposed by Fredlund and Hasan [19]. This theory introduces the nonlinear governing equations describing independent flows of air and water in unsaturated soil deposits. Later, Dakshanamurthy and Fredlund [20], and Dakshanamurthy et al. [21] expanded the existing equations to two-dimensional (2D) and three-dimensional (3D) models, respectively. This set of theories has inspired a large number of recent studies, of which analytical and numerical models have been conducted in the Cartesian

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List of symbols

A	constant of eigenfunction with respect to the radial domain
B	constant of eigenfunction with respect to the radial domain
C_a	interactive constant associated with the air phase
C_w	interactive constant associated with the water phase
$c_{v_r}^{a}$	coefficient of consolidation with respect to the air phase in the radial domain
$c_{v_z}^{a}$	coefficient of consolidation with respect to the air phase in the vertical domain
$c_{v_r}^{w}$	coefficient of consolidation with respect to the water phase in the radial domain
$c_{v_z}^{w}$	coefficient of consolidation with respect to the water phase in the vertical domain
g	gravitational constant
H	thickness of the soil stratum
i	integer for the Fourier Bessel series
j	integer for the Fourier sine series
k_{a_r}	coefficient of permeability for the air phase in the radial domain
k_{a_z}	coefficient of permeability for the air phase in the vertical domain
k_{w_r}	coefficient of permeability for the water phase in the radial domain
k_{w_z}	coefficient of permeability for the water phase in the vertical domain
M	molecular mass of the air phase
m_1^a	coefficient of volume change of the air phase with respect to the change of net stress
m_2^a	coefficient of volume change of the air phase with respect to the change of suction
m_1^s	coefficient of volume change of the soil with respect to the change of net stress
m_2^s	coefficient of volume change of the soil with respect to the change of suction
m_1^w	coefficient of volume change of the water phase with respect to the change of net stress
m_2^w	coefficient of volume change of the water phase with respect to the change of suction
n	porosity
q_0	external loading
R	universal air constant
S	center to center spacing in the vertical drain system
S_r	degree of saturation
s	matric suction
T	time factor
$T_a(t)$	generalized Fourier coefficients varying with time with respect to the air phase
$T_w(t)$	generalized Fourier coefficients varying with time with respect to the water phase
t	elapsed time
$\overline{U}$	average degree of consolidation
u_a	excess pore-air pressure
$u_{a,t}$	first order of partial differential equation (PDE) of air with respect to time
$u_{a,r}$	first order of partial differential equation (PDE) of air with respect to radius
$u_{a,rr}$	second order of partial differential equation (PDE) of air with respect to radius
$u_{a,zz}$	second order of partial differential equation (PDE) of air with respect to depth
u_{atm}	atmospheric pressure
u_a^0	initial excess pore-air pressure at the soil surface
u_a^z	initial excess pore-air pressure at depth z
$\bar{u}_a$	average excess pore-air pressure
u_w	pore-water pressure
$u_{w,t}$	first order of partial differential equation (PDE) of water with respect to time
$u_{w,r}$	first order of partial differential equation (PDE) of water with respect to radius
$u_{w,rr}$	second order of partial differential equation (PDE) of water with respect to radius
$u_{w,zz}$	second order of partial differential equation (PDE) of water with respect to depth
u_w^0	initial excess pore-water pressure at the soil surface
u_w^z	initial excess pore-water pressure at depth z
$\bar{u}_w$	average excess pore-water pressure
V_0	initial volume of the soil element
V_a	volume of air within soil element
V_w	volume of water within soil element
$R_a(r)$	eigenfunction with respect to the excess pore-air pressure for domain r
$R_w(r)$	eigenfunction with respect to the excess pore-water pressure for domain r
r	investigated radius
r_e	radius of zone of influence
r_w	radius of the drain well
$Z_a(z)$	eigenfunction with respect to the excess pore-air pressure for domain z
$Z_w(z)$	eigenfunction with respect to the excess pore-water pressure for domain z
z	investigated depth
γ_w	water unit weight
Δs	suction change
ε_v	total volumetric strain
$\bar{\varepsilon}_v$	average total volumetric strain
ζ_a	parameter controlling the distribution of the initial excess pore-air pressure
ζ_w	parameter controlling the distribution of the initial excess pore-water pressure
Θ	absolute temperature in Kelvin
θ	polar angle
θ°	temperature in degree celsius
λ_a^{ij}	separation constant with respect to the air phase
λ_w^{ij}	separation constant with respect to the water phase
μ^j	eigenvalue for vertical boundary condition (i.e. PTIB and PTPB)
ξ^i	eigenvalue for radial boundary condition
σ_r	total stress in r-domain
σ_z	total stress in z-domain
σ_θ	total stress in θ-domain

coordinate system. Further unsaturated soil studies include 1D consolidation settlement of a single layer unsaturated soil [22–27], consolidation of multi-layered soil [28], consolidation for viscoelastic soil [29], and 2D plane strain consolidation problems [30–32]. There have been, however, few attempts to model axisymmetric unsaturated consolidation particularly with analytical approaches. Among pioneer studies, Conte [33] introduced the finite element technique to obtain a solution for the coupled consolidation under the plane strain and axisymmetric conditions. Qin et al. [34] dealt with the drain well consolidation problem in unsaturated soils using the modified Bessel functions and the Laplace transformation. On the other hand, Zhou and Tu [35],