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Research Paper

Bearing capacity of strip footings on unsaturated soils by the slip line theory

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ABSTRACT

The bearing capacity of strip footings on unsaturated soils is studied using slip line theory. The suction profiles considered are non-uniform with depth and correspond to vertical flow of water by infiltration or evaporation. The slip line theory assumes a plastic equilibrium state of a Mohr–Coulomb soil in which suction influences are included using the effective stress concept. This paper shows the similar and independent effects of cohesion and the contribution of suction to the effective stress in the governing equations. It shows that the influence of a non-uniform suction profile on bearing capacity is significant, and the depth to the ground water table and the footing width have significant roles in how much suction influences the bearing capacity. By approximating the contribution of suction to the effective stress as a function that varies linearly with depth, the effect of suction on bearing capacity is represented in dimensionless form in separate charts for smooth and rough footings. Using the charts the bearing capacity can be determined for any combination of friction angle, footing width, surcharge, soil unit weight and linear profiles of cohesion and the contribution of suction to the effective stress. An example is given which highlights the significant influence suction can have. These charts also permit assessment of bearing capacity changes that may occur when changes to suction are expected, due to seasonal fluctuations of soil moisture, drought or flooding.

1. Introduction

Foundations are ubiquitous to geotechnical engineering. Many of the currently used design theories for estimating ultimate bearing capacities of foundations were developed using limit equilibrium methods and the slip line theory. They are mostly applied to foundation soils that are fully saturated or dry. Shallow footings, however, operate in the upper fewer metres of the ground, often above the water table where foundation soils are variably saturated and may never become fully saturated. Although the bearing capacities of shallow foundations on unsaturated soils can be assessed by existing design theories in certain conditions, the approach and the conditions when they apply have not been clearly outlined in the literature.

An estimated 60% of the world's population lives in regions where the surface soils (to a depth of several metres) are unsaturated [1]. Soils which are unsaturated have an internal suction that increases particle to particle contact forces and thus soil strength and stiffness. However, studies on how to incorporate suction into a bearing capacity are limited.

The significant influence that suction can have on bearing capacity has been recognised in the literature. Meyerhof [2] attributed an increase in a sand's bearing capacity to the presence of suction above the water table and to a higher effective unit weight there. Meyerhof [2] and Vesic [3] recommended suction be treated as an apparent cohesion and shallow foundations be designed based on the highest possible ground water table. Vesic [3] suggested that when the distance between the ground water table and the base of the foundation is less than the foundation width then the soil unit weight should be modified to account for suction above the water level. More recently Oloo et al. [4] and Costa et al. [5] incorporated suction into Terzaghi-type bearing capacity calculations by treating it as an apparent cohesion that was constant with depth. A suction related term, in addition to cohesion, contributed to bearing capacity when scaled by the bearing capacity factor N_c. Xu [6] observed in experiments a suction dependant increase in bearing capacity in unsaturated expansive soils. Jahanandish et al. [7] accounted for this increase in an analysis using zero extension line theory. Again suction was treated as an apparent cohesion constant with depth that contributed to bearing capacity through N_c.

Never before has the effect of a non-uniform suction profile with depth been considered in a bearing capacity analysis. Suction

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List of notations

Greek letters	
γ_t, γ_w	soil unit weight and water unit weight
δ'	interface friction angle
η, ξ	stress characteristic curves
θ	angle between the vertical axis and the major principal stress direction
μ	angle between η, ξ curves and the major principal stress direction
λ	inverse of s_e
σ_1, σ_3	major and minor principal stresses
σ_m	mean stress
σ_{zz}, σ_{xx}	normal stress in the z and x directions
σ_{xz}, σ_{zx}	shear stress
φ'	soil friction angle
χ	effective stress parameter
Roman letters	
B	width of strip footing
c'	soil cohesion
c'_0	cohesion at soil surface
H	depth to ground water table
k, k_s	unsaturated and saturated hydraulic conductivity
K_c, $K_{\chi s}$	rate of change of cohesion and χs with depth
L	length of surcharge
N_c, N_γ	bearing capacity factors
q	steady state flow rate
q_u, Q_u	ultimate bearing capacity (pressure and load)
q_s	surcharge
s	soil suction
s_e	suction value separating saturated from unsaturated states
s_{ae}, s_{ex}	air entry suction, air expulsion suction
u_a, u_w	pore air and pore water pressures
V	dimensionless vertical bearing capacity
z	vertical direction (positive downwards, zero at the soil surface)
x	horizontal direction (positive to the right, zero at the right edge of the footing)

may be highest at very shallow depths where soil interacts with the environment and evaporation may occur. But suction may decay with depth and vanish all together at the ground water table. This non-uniformity introduces uncertainties into how suction may influence bearing capacity and to what extent it may be relied upon during the service life of the footing. One aim of this paper is to study the influence of non-uniform suction profile, set up through steady state evaporation and infiltration, on the bearing capacity of a surface strip footing. A second aim is to show when and how established design theories may be extended and used to estimate the ultimate bearing capacities of shallow foundations on unsaturated soils.

2. Governing equations

The Mohr–Coulomb failure criterion is adopted in this paper. Suction contributes to the effective stress in a soil and therefore to its shear strength. The influence of suction may make the unsaturated soil non-homogeneous [7–9], evident by a spatial derivative of the suction contribution to strength in the governing equations. Advanced numerical tools can be used to solve the governing equations, e.g. the finite element method and computational limit analysis. Here the method of characteristics and slip line theory [10] are used.

The coordinates system adopted in this paper is $z \equiv$ vertical direction (positive downwards, zero at the soil surface) and $x \equiv$ horizontal direction (positive to the right, zero at the right edge of the footing) (Fig. 1).

2.1. Effective stress and failure criterion

The governing equations are formulated based on the effective stress concept for unsaturated soils [11]. The effective stress (σ') is defined as:

$$\sigma' = \sigma - u_a + \chi(u_a - u_w) \tag{2.1.1}$$

where $\sigma \equiv$ total stress, $u_a \equiv$ pore air pressure, $u_w \equiv$ pore water pressure and $\chi \equiv$ effective stress parameter. Eq. (2.1.1) can be rewritten as:

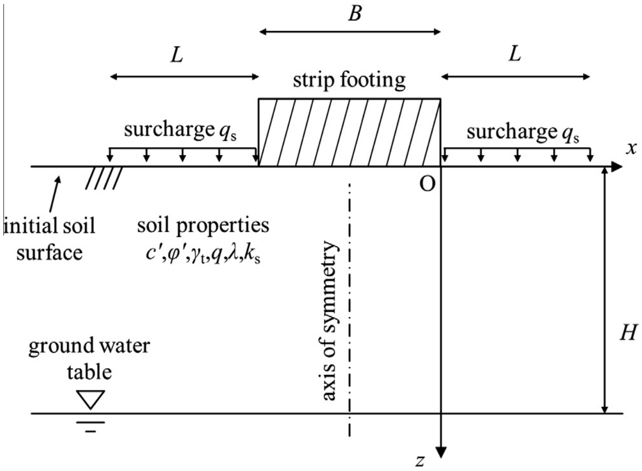

Fig. 1. Problem geometry and parameters.

$$\sigma' = \sigma_n + \chi s \tag{2.1.2}$$

where $s = u_a - u_w \equiv$ suction and $\sigma_n = \sigma - u_a \equiv$ net stress, that is the total stress in excess of pore air pressure. When pore air pressure is equal to atmospheric pressure and is taken as the pressure datum, the net stress and total stress become the same. The subscript n may then be removed to simplify notation ($\sigma' = \sigma + \chi s$). This will be done here.

The expression for χ when the hydraulic state of the soil is located on the main drying or the main wetting branch of the soil–water characteristic curve is [12]:

$$\begin{cases} \chi = 1 & \text{for } s \leqslant s_e \\ \chi = \left(\frac{s}{s_e}\right)^{-0.55} & \text{for } s > s_e \end{cases} \tag{2.1.3}$$

where $s_e \equiv$ suction value separating saturated from unsaturated states ($s_e = s_{ae} \equiv$ air entry suction and applies when the hydraulic state is on the main drying branch, $s_e = s_{ex} \equiv$ air expulsion suction and applies when the hydraulic state is on the main wetting branch). Eq. (2.1.3) is appealing because it is simple and has been