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Research Paper

Mixed-mode fracture modeling with smoothed particle hydrodynamics

Thomas Douillet-Grellier [a,b,*], Bruce D. Jones [a], Ranjan Pramanik [a], Kai Pan [a], Abdulaziz Albaiz [a], John R. Williams [a]

[a] MIT Geonumerics – Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA
[b] Total E&P, Pau, France

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ABSTRACT

In this work, we describe a coupled Drucker–Prager and Grady–Kipp SPH framework in order to simulate mode I, mode II and mixed mode failure under the same formulation. This framework is then applied to study failure in uniaxial compression of gypsum samples containing a single angled flaw.

To validate the model, results are compared with the experimental analysis and shows good agreement, where fracture initiation positions and angles are well represented. This trend of agreement continues beyond initiation, through to propagation, and finally post-failure behavior.

The methodology and results shown here describe a powerful tool for study of fracture mechanics.

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1. Introduction

Meshless methods such as the Smoothed Particle Hydrodynamics (SPH) method have been applied in recent years to study problems in rock mechanics. First developed to study problems in astrodynamics by Gingold and Monaghan [1], and Benz et al. [2], the method has since been successfully applied to a broad range of problems. These include, but are not limited to, elastic flow [3], fluid flow [4,5], impact problems [6], heat transfer problems [7], multiphase flow [8,9], geophysical flow [10], fluid–structure interaction [11,12] and post-failure of cohesive and non-cohesive soils [13].

The first application of SPH to solid mechanics was carried out by Libersky and Petschek [14]. Their work was subsequently extended to simulation of the fracture process in brittle solids by Benz and Asphaug [15]. Randles and Libersky [16] and then Gray et al. [17] have since made significant improvements in extending SPH to elastic dynamics. More recently, Douillet-Grellier et al. [18] proposed a new approach for stress based boundary conditions in SPH, which has been validated for elastic problems in solid mechanics. Finally, Liu and Liu [19] and Monaghan [20] both give exhaustive reviews of recent developments within the SPH community.

SPH is based upon a formulation where multiple crack initiations and propagations may emerge naturally from a well formulated constitutive model. Due to the meshless nature of the method, no remeshing is required to simulate large deformations. Similar to more traditional methods such as the finite element method, material properties may be specified *a priori*, requiring no calibration. The challenge of accurate fracture modeling therefore lies in the choice and implementation of suitable constitutive models to capture both shear and tensile failure.

A number of authors have pursued the Drucker–Prager plasticity model in order to capture plasticity and shear failure in SPH models. The first implementation of Drucker–Prager plasticity in an SPH framework was carried out by Bui et al. [13], who used this methodology to study soil collapse and slope stability. This approach has since been used to study failure due to compression in a Brazilian test [21,22], failure in jointed media [23], large deformation in granular materials [24], and the shear box laboratory experiment in a multiscale framework [25].

For tensile failure, the Grady–Kipp damage model can be incorporated into an SPH framework as proposed by Benz and Asphaugh [26,15] and validated for failure in solid bars loaded in tension. Use of the Grady–Kipp damage model has proved successful for simulation of high speed impacts [15,27,28]. In addition to impact problems, the Grady–Kipp damage model has also been applied to simulation of failure in magma chambers [29], as well as the penny shaped crack problem [30].

The accuracy of the Drucker–Prager and Grady–Kipp constitutive models have been well demonstrated in the literature. This

* Corresponding author at: MIT Geonumerics – Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, 77 Massachusetts Avenue, 02139, Cambridge, MA, USA.

E-mail address: thomasdg@mit.edu (T. Douillet-Grellier).

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work is concerned with SPH simulation of mixed-mode fracture during uniaxial compression of gypsum samples including preexisting flaws. Thus, we begin with a description of the Drucker–Prager and Grady–Kipp constitutive models, followed by discussion on coupling of these two constitutive models to capture both shear and tensile failure. The coupled Drucker–Prager and Grady–Kipp framework described herein is then validated against the experimental results of Wong [31].

2. Governing equations

An SPH discretization begins with governing equations for mass and momentum conservation equations in a Lagrangian system, which are given as

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\boldsymbol{v} \tag{1}$$

$$\frac{D\boldsymbol{v}}{Dt} = \frac{1}{\rho}\nabla\cdot\boldsymbol{\sigma} + \mathbf{g} \tag{2}$$

where ρ is the density; $\boldsymbol{v}$ is the velocity; $\boldsymbol{\sigma}$ is the stress tensor; $\mathbf{g}$ is the external body force per unit mass and D/Dt denotes the material derivative following the motion.

To close the system of equations given by Eqs. (1) and (2) a constitutive relation for evaluation of the stress tensor must be given. This begins by splitting the stress tensor into deviatoric and hydrostatic parts.

$$\boldsymbol{\sigma} = \frac{tr(\boldsymbol{\sigma})}{3}\boldsymbol{I} + \boldsymbol{s} = -p\boldsymbol{I} + \boldsymbol{s} \tag{3}$$

in which p is the hydrostatic pressure, and $\boldsymbol{s}$ is the second order deviatoric stress tensor.

With this definition the hydrostatic pressure is treated in the same manner as fluid pressure in an SPH fluid framework. A density based state *equation of state* may therefore be used [3,16]. However, Bui et al. proposed that hydrostatic pressure may instead be calculated directly from the rock constitutive equation by the standard definition of mean stress [13], where this approach is adopted in this work.

The above set of governing equations are then solved through discretization to a set of particles, by exploiting an interpolating kernel.

2.1. Constitutive model

The components of the strain rate tensor are given as

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2}(\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^{\mathsf{T}}) \tag{4}$$

As the stress tensor in (3), the strain rate tensor is expressed as a combination of a hydrostatic and deviatoric parts.

$$\dot{\boldsymbol{\varepsilon}} = \left(\dot{\boldsymbol{\varepsilon}} - \frac{tr(\dot{\boldsymbol{\varepsilon}})}{3}\boldsymbol{I}\right) + \frac{tr(\dot{\boldsymbol{\varepsilon}})}{3}\boldsymbol{I} = \dot{\boldsymbol{\varepsilon}}_d + \dot{\boldsymbol{\varepsilon}}_v \tag{5}$$

where $\dot{\boldsymbol{\varepsilon}}_d$ and $\dot{\boldsymbol{\varepsilon}}_v$ are the deviatoric and volumetric parts of the elastic strain rate tensor, which is calculated by the generalized Hooke's law as

$$\dot{\boldsymbol{\sigma}} = 2G\dot{\boldsymbol{\varepsilon}}_d + K\dot{\boldsymbol{\varepsilon}}_v \tag{6}$$

in which G and K are the shear and bulk moduli respectively. A yield function, $F(\boldsymbol{\sigma},\boldsymbol{q})$, defines the elastic domain as the set

$$\chi = \{\boldsymbol{\sigma}|F(\boldsymbol{\sigma},\boldsymbol{q}) < 0\} \tag{7}$$

where $\boldsymbol{q}$ is the component of internal variables associated with the phenomenon of softening. It is convenient to define the flow rule in terms of flow (or plastic) potential, $Q(\boldsymbol{\sigma},\boldsymbol{q})$. Then the flow vector, $\boldsymbol{N}$, is obtained as

$$\boldsymbol{N} = \frac{\partial Q(\boldsymbol{\sigma},\boldsymbol{q})}{\partial\boldsymbol{\sigma}} \tag{8}$$

In this work, the plastic potential function does not coincide with the yield function, $F(\boldsymbol{\sigma},\boldsymbol{q})$, the flow rule is then called a non-associated flow rule. The general elasto-plastic constitutive model is given by

$$\dot{\boldsymbol{\varepsilon}}_e = \dot{\boldsymbol{\varepsilon}} - \dot{\lambda}\boldsymbol{N} \tag{9}$$

$$\dot{\boldsymbol{q}} = -\dot{\lambda}\frac{\partial Q(\boldsymbol{\sigma},\boldsymbol{q})}{\partial\boldsymbol{q}} \tag{10}$$

where $\dot{\boldsymbol{\varepsilon}}_e$ is the elastic strain rate tensor. This is taken together with the loading–unloading conditions

$$\dot{\lambda} \geqslant 0, \quad F \leqslant 0, \quad \dot{\lambda}F = 0 \tag{11}$$

where $\dot{\lambda}$ is the plastic consistency parameter.

The above initial value problem (9)–(11) should be solved numerically for the set of softening internal variables and the plastic multiplier of each SPH particle. This procedure will be discussed in more detail in Section 4.1.1.

3. The SPH method

An SPH model consists of a set of points with fixed volume, which possess material properties and interact with all neighboring particles by a weighting function (or smoothing kernel) [1]. A particles support domain, Λ, is given by its smoothing length, h, which is in turn the radius of the smoothing kernel. To obtain the value of a function at a given particle location, values of that function are found by taking a weighted (by the smoothing function) interpolation from all particles within the given particles support domain. Analytical differentiation of the smoothing kernel is used to find gradients of this function. Detailed concepts and descriptions of this method are given by Monaghan [3].

To begin, we define the kernel estimation

$$A(\boldsymbol{x}) = \int_{\Omega} A(\boldsymbol{x}')W(\boldsymbol{x}-\boldsymbol{x}',h)d\boldsymbol{x}, \quad \forall\ \boldsymbol{x}\in\Omega\subset\mathbb{R}^d \tag{12}$$

where A is a vector function of the position vector $\boldsymbol{x}$, Ω is the volume of the integral containing the point $\boldsymbol{x}$, and $W(\boldsymbol{x}-\boldsymbol{x}',h)$ is the smoothing kernel. The interpolated value of a function A at the position $\boldsymbol{x}_a$ of particle a can be expressed using SPH smoothing as

$$A(\boldsymbol{x}_a) = \sum_{b\in\Lambda_a} A_b\frac{m_b}{\rho_b}W(\boldsymbol{x}_a-\boldsymbol{x}_b,h). \tag{13}$$

in which m_b and ρ_b are the mass and the density of neighboring particle b. $\Lambda_a = \{b\in N|\ |\boldsymbol{x}_a-\boldsymbol{x}_b|\leqslant\kappa(h_a+h_b)/2\}$ is the set of particles which are neighbors of particle a and lie within its defined support domain. κ depends on the choice of the kernel, it is equal to 2 for the cubic spline kernel function used in this paper. The gradient of the function A at the position of particle a is evaluated by differentiating the smoothing kernel W given in Eq. (13) as

$$\nabla A(\boldsymbol{x}_a) = \sum_{b\in\Lambda_a} A_b\frac{m_b}{\rho_b}\nabla_a W(\boldsymbol{x}_a-\boldsymbol{x}_b,h). \tag{14}$$

In practice, Libersky et al. find that by exploiting the symmetric properties of the kernel, a more accurate formulation is found [32] as

$$\nabla A(\boldsymbol{x}_a) = \sum_{b\in\Lambda_a} (A_b-A_a)\frac{m_b}{\rho_b}\nabla_a W(\boldsymbol{x}_a-\boldsymbol{x}_b,h). \tag{15}$$

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