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Research Paper

Advanced Boundary Element analysis of geotechnical problems with geological inclusions

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ABSTRACT

In this work a novel approach is presented for the Boundary Element analysis of problems in geomechanics. Firstly, Non-Uniform Rational B-Spines (NURBS) are used for the description of the geometry and for the approximation of the unknowns. This results in a significant decrease in the number of parameters used for an accurate description of the geometry as well as a decrease in the number of degrees of freedom required for good quality results. Secondly, NURBS are also used for the description of the geometry of geological inclusions, which can have properties different to the rock mass and can experience inelastic behavior.

After a short introduction to the theory, some details of implementation are shown. On test examples, involving elastic homogeneous domains, it is first shown that the method delivers accurate results with fewer parameters and number of unknowns as compared with conventional analysis. Solutions are compared to either known solutions or with conventional BEM analyses. Geological inclusions are introduced next and results of test examples are compared with Finite Element analyses. Finally a practical example is shown.

1. Introduction

The Boundary Element Method (BEM) is ideally suited for the analysis of problems in geomechanics as it can easily consider infinite and semi-infinite domains, since the radiation condition is implicitly fulfilled. In the case of elastic, homogeneous domains only boundary integrals appear, and the solution involves a discretization of the boundary, thereby reducing the analysis effort by an order of magnitude.

However, to analyze real problems in geomechanics the consideration of heterogeneous and inelastic ground conditions is essential. The BEM can be extended to analyze these problems, but additional volume integrals appear. The solution requires the discretization of a volume, the attractiveness of the method is considerably reduced. However, the volume integrals only cover the part of the domain that has different material properties or behaves in an inelastic way. Currently the most popular method is to use internal cells for the volume discretization. Cells are basically identical to Finite Elements but the main difference is that no additional degrees of freedom are introduced, as their only purpose is to evaluate the volume integral. The requirement for an additional volume discretization seems to have severely restricted the application of the BEM in geomechanics, with the Finite Element or similar domain methods dominating. In this paper a novel approach will be presented that does not require the generation of a cell mesh.

Isogeometric analysis [1] has gained significant popularity in the last decade because of the fact that geometry data can be taken directly from Computer Aided Design (CAD) programs, potentially eliminating the need for mesh generation. NURBS basis functions, that are used for the definition of the geometry, are able to describe certain geometries such as arcs exactly. Therefore, as will be shown, the number of parameters, required to accurately define geometry, can be reduced significantly. As will be shown, NURBS patches can also be used to define geological inclusions, opening the way to use geological information directly from CAD programs.

2. The BEM with volume effects

To apply the BEM to heterogeneous and inelastic problems, so called body force effects have to be included. Using the theorem of Betti as explained in [2], the boundary integral equation with

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body forces acting in a sub-volume V_0 can be written in incremental form and in matrix notation as (see Fig. 1):

$$\mathbf{c}\dot{\mathbf{u}}(\mathbf{y}) = \int_S \mathbf{U}(\mathbf{y},\mathbf{x})\dot{\mathbf{t}}(\mathbf{x})\,dS + \int_{S_0} \mathbf{U}(\mathbf{y},\bar{\mathbf{x}})\dot{\mathbf{t}}_0(\bar{\mathbf{x}})\,dS_0 - \int_S \mathbf{T}(\mathbf{y},\mathbf{x})\dot{\mathbf{u}}(\mathbf{x})\,dS + \int_{V_0} \mathbf{U}(\mathbf{y},\bar{\mathbf{x}})\dot{\mathbf{b}}_0(\bar{\mathbf{x}})\,dV_0 \tag{1}$$

where $\mathbf{c}$ is a free term, $\mathbf{U}(\mathbf{y},\mathbf{x})$ and $\mathbf{T}(\mathbf{y},\mathbf{x})$ are matrices containing fundamental solutions (Kernels) for the displacements and tractions at a point $\mathbf{x}$ due to a unit force at a point $\mathbf{y}$ [3], $\dot{\mathbf{u}}(\mathbf{x})$ and $\dot{\mathbf{t}}(\mathbf{x})$ are increments of the displacement and traction vectors on the surface S, defining the problem domain. $\dot{\mathbf{b}}_0(\bar{\mathbf{x}})$ are increments of body force inside the inclusion and $\dot{\mathbf{t}}_0(\bar{\mathbf{x}})$ are increments of tractions related to the body force acting on surface S_0 bounding V_0.

The integral equations can be solved for the unknowns $\mathbf{u}$ or $\mathbf{t}$ by discretization. As in the majority of previous work on the isogeometric BEM [4–10] we use the collocation method, i.e. we write the integral equations for a finite number, N, of source or collocation points $\mathbf{y}_n$:

$$\mathbf{c}\dot{\mathbf{u}}(\mathbf{y}_n) = \int_S \mathbf{U}(\mathbf{y}_n,\mathbf{x})\dot{\mathbf{t}}(\mathbf{x})\,dS + \int_{S_0} \mathbf{U}(\mathbf{y}_n,\bar{\mathbf{x}})\dot{\mathbf{t}}_0(\bar{\mathbf{x}})\,dS_0 - \int_S \mathbf{T}(\mathbf{y}_n,\mathbf{x})\dot{\mathbf{u}}(\mathbf{x})\,dS + \int_{V_0} \mathbf{U}(\mathbf{y}_n,\bar{\mathbf{x}})\dot{\mathbf{b}}_0(\bar{\mathbf{x}})\,dV_0 \tag{2}$$

with $n = \{1,\ldots,N\}$.

For the numerical evaluation of the surface integrals over S we divide the boundary into patches and use a geometry independent field approximation approach for each patch, i.e. we use different basis functions for the description of the geometry and for the field values:

$$\mathbf{x}^e = \sum_{k=1}^{K} N_k(\mathrm{u}) \cdot \mathbf{x}_k^e \tag{3}$$

$$\mathbf{u}^e = \sum_{k=1}^{K^d} N_k^d(\mathrm{u}) \cdot \mathbf{u}_k^e \tag{4}$$

$$\mathbf{t}^e = \sum_{k=1}^{K^t} N_k^t(\mathrm{u}) \cdot \mathbf{t}_k^e \tag{5}$$

In the above the superscript e refers to the number of the patch, N_k, N_k^d, N_k^t are NURBS basis functions of the local coordinate u for describing the geometry $\mathbf{x}^e$, displacements $\mathbf{u}^e$ and tractions $\mathbf{t}^e$, $\mathbf{x}_k^e$ specify the location of control points, $\mathbf{u}_k^e, \mathbf{t}_k^e$ are parameter values and K, K^d, K^t specify the number of parameters for each patch.

For an excavation problem for example the following system of equations can be assembled:

$$[\mathbf{T}]\{\mathbf{u}\} = \{\mathbf{F}\} + \{\mathbf{F}\}_0 \tag{6}$$

where $[\mathbf{T}]$ is an assembled matrix with coefficients related to Kernel $\mathbf{T}$ and $\{\mathbf{u}\}$ is a vector that collects all displacement components on points $\mathbf{y}_n$. $\{\mathbf{F}\}$ is a vector related to the applied tractions due to excavation and $\{\mathbf{F}\}_0 = \{\mathbf{F}\}_0^{S_0} + \{\mathbf{F}\}_0^{V_0}$ is the right hand side related to the body force effects, i.e. related to the integrals over S_0 and V_0 in Eq. (2). Details of the implementation of the isogeometric BEM for elastic homogeneous domains can be found in [3,11].

3. NURBS basis functions

A detailed treatise on NURBS basis functions is presented in [3], here only a short explanation is given. NURBS or Non-uniform rational B-splines are an extension of classical B-splines. To define B-splines we start with a *knot vector*. This is a vector containing a series of non-decreasing values of the local coordinate:

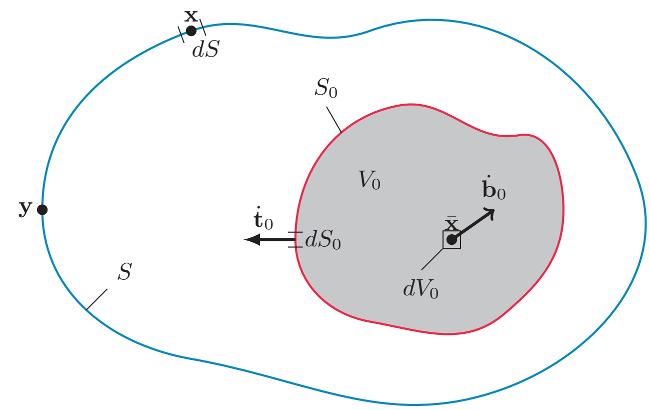

Fig. 1. Explanation of the derivation of the integral equation with volume effects.

$$\varXi = (\mathrm{u}_0 \quad \mathrm{u}_1 \quad \cdots \quad \mathrm{u}_N) \tag{7}$$

We define the entries in the vector as *knots*. With the knot vector a recursive formula is applied. First we compute the functions for order $p = 0$ (constant) and for $i = 0,\ldots,N$.

$$N_{i,0}(\mathrm{u}) = \begin{cases} 1 & \text{if} \quad \mathrm{u}_i \leqslant \mathrm{u} < \mathrm{u}_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Higher order basis functions are defined by referencing lower order functions:

$$N_{i,p}(\mathrm{u}) = \frac{\mathrm{u} - \mathrm{u}_i}{\mathrm{u}_{i+p} - \mathrm{u}_i} N_{i,p-1}(\mathrm{u}) + \frac{\mathrm{u}_{i+p+1} - \mathrm{u}}{\mathrm{u}_{i+p+1} - \mathrm{u}_{i+1}} N_{i+1,p-1}(\mathrm{u}) \tag{9}$$

NURBS basis functions are obtained by including weights, w_i:

$$R_{i,p}(\mathrm{u}) = \frac{N_{i,p}(\mathrm{u})\,\mathrm{w}_i}{\sum_{j=0}^{I} N_{j,p}(\mathrm{u})\,\mathrm{w}_j} \tag{10}$$

4. Geometry description with NURBS

NURBS are ideally suited for the description of geometry (for example they are able to describe circular arcs exactly) and this is one of the main reasons they are used by the CAD community. The main difference to commonly used Lagrange polynomials, is that the concept of nodal points is replaced by a concept of control points, which do not always lie on the curve.

As an example we show the description of the geometry of an NATM tunnel, where the design shape is specified by arcs (center, radius and extent) as shown in the tables in Fig. 2. One half of the tunnel can be described with 1 NURBS patch of order 2 and only 7 control points.

The control point coordinates and weights can be computed from arc centers, radii and start/end angles using a simple formula (see [3]) and are given by:

x	y	weight
0.0	5.65	1.0
4.55	5.65	0.707
4.55	1.1	1.0
4.55	-0.97	0.82
2.61	-1.67	1.0
1.33	-2.04	0.99
0.0	-2.04	1.0