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Research Paper

Probabilistic stability analysis of simple reinforced slopes by finite element method

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ABSTRACT

Probabilistic slope stability analyses of simple geosynthetic reinforced soil slopes were carried out using the shear strength reduction method in combination with the finite element method (FEM). An existing open-source FEM code was modified to include bar elements to model horizontal layers of geosynthetic reinforcement. Analysis results using the modified code (mFEM) demonstrate that large reductions in probability of failure can be realized by adding geosynthetic reinforcement layers to constructed slopes. The modified code was also used to investigate the effect of variability of soil friction angle on probabilistic outcomes for constructed unreinforced and reinforced purely frictional soil slopes.

1. Introduction

Geosynthetic-reinforced slopes (and embankments) are widely used in geotechnical engineering. Since the early 1980s, conventional deterministic limit equilibrium methods (LEMs) for unreinforced slopes have been modified to include the stabilizing force (or moment) contribution of geosynthetic reinforcement layers in constructed slopes and embankments. These methods include circular slip [26], log-spiral [29] and two-part wedge [34,8] approaches. The margin of safety is computed as a single-value critical factor of safety for the reinforced slope.

It has been frequently demonstrated in the literature that two nominally identical natural slopes can have the same factor of safety based on conventional deterministic factor of safety analysis methods but have very different probabilities of failure due to random and spatial variability of the soil properties [38,13,19,9,22]. While uncertainty in the properties of engineered fills used in constructed unreinforced and reinforced slopes are likely low compared to values reported for natural *in situ* soil materials (e.g. [33,11,15], amongst many others), the influence of soil material variability on margins of safety expressed in probabilistic terms for reinforced slopes and embankments has received little attention.

Kitch [26] carried out probabilistic analyses of two reinforced slope examples with reinforcement layouts initially selected using design charts based on deterministic limit equilibrium methods. Low and Tang [30] proposed a limit equilibrium stability model for reinforced embankments on soft ground and a practical reliability evaluation procedure. However, the LEM approach used in both studies has the disadvantage that the type of critical failure surface must be assumed *a priori* (e.g. circular, non-circular or bi-linear) and an assumption must be made regarding the magnitude and distribution of available stabilizing reinforcement tensile forces.

More recently, a probabilistic analysis technique called the Random Finite Element Method (RFEM) has been developed by Griffiths and Fenton [19] to conduct probabilistic stability analysis of slopes with spatial variability of soil properties based on random field theory. In the limit of infinite spatial correlation length the Random Finite Element Method can be understood to be the FEM with only random variability of soil properties. Conventional deterministic FEM slope stability analyses are also possible by assigning only constant values for the soil parameters. The shear strength reduction method is used to compute the factor of safety in each analysis. An advantage of the combined FEM-shear strength reduction method is that this approach allows the program to search out

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Abbreviations: FEM, finite element method; LEM, limit equilibrium method; mFEM, modified finite element method; pLEM, probabilistic limit equilibrium method; RFEM, random finite element method.

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the critical failure mechanism without constraints imposed by fixed failure geometry assumptions [20].

In the current study an existing open-source FEM program described by Griffiths and Fenton [19] and Fenton and Griffiths [15] was expanded to allow for probabilistic analysis of reinforced slope cases. Although the original and expanded code can consider spatial variability of soil properties, in this investigation only random soil variables were considered in the probabilistic analyses. Hereafter, the original code is referred to as the FEM code or program and the modified code is referred to as the mFEM code or program to avoid confusion with the random field theory capability of the original source program.

The original (unmodified) FEM code was first validated by comparing results of conventional and probabilistic analysis of unreinforced slope cases. In this paper, probabilistic slope stability analyses using LEM methods together with probability theory are referred to as probabilistic limit equilibrium methods (pLEMs). Deterministic analysis results using the (modified) mFEM source code (i.e. without random soil properties) were also compared to results of FEM analysis of reinforced slopes using a commercial FEM software package and assuming purely frictional soils. Good agreement was demonstrated between programs for the prediction of reinforcement strains giving confidence that implementation of the reinforcement capability in the mFEM code is correct. Predictions of factor of safety and failure modes of reinforced slopes using the mFEM and LEM (Bishop's Simplified Method with addition of reinforcement forces) were also carried out. Differences in results were attributed to the treatment of stabilizing forces in LEM analyses. A strategy proposed by Hammah et al. [21] is used to eliminate discrepancies between LEM and the coupled FEM-strength reduction method for the calculation of factor of safety for simple homogenous soil slopes.

The utility of the expanded mFEM code for probabilistic analysis of the factor of safety for simple soil slopes with frictional soils is demonstrated using a number of unreinforced and reinforced slope examples. In the probabilistic study, the influence of variability of soil friction angle (ϕ) and unit weight (γ) on the probabilistic outcomes of both unreinforced and reinforced purely frictional soil slopes is investigated. The effect of variability in γ and cross-correlation between ϕ and γ on probability of failure is investigated and (as expected) shown to have no effect.

2. Verification of FEM code for unreinforced slopes

2.1. General

With the exception of modifications for reinforced slopes, the open-source FEM code (mrslope2d) described by Fenton and Griffiths [15] and available at "http://courses.engmath.dal.ca/rfem/" was used in the current study. The deterministic slope stability analysis part of the code (Program 6.4, [37] is based on the shear strength reduction method. This program is for two-dimensional slope stability analysis of unreinforced slopes with elastic-perfectly plastic soils governed by the Mohr–Coulomb failure criterion. Eight-node quadrilateral elements are used. The bottom of the slope foundation is fixed in both horizontal and vertical directions. The vertical boundaries on both sides are fixed in the horizontal direction. The gravity "turn-on method" is used in Program 6.4 (see [20]).

2.2. Comparison of FEM with pLEM approaches (unreinforced slopes)

2.2.1. Cohesive soil slopes

Results using the FEM method (i.e. original code without reinforcement) are first compared to pLEM results where the latter are taken from probabilistic stability design charts for simple unreinforced purely cohesive soil slopes developed by Javankhoshdel and Bathurst [22]. They expressed Taylor's slope stability equation [39] using random variable notation as follows:

$$\bar{F}_s = \frac{\mu_{su}}{\mu_\gamma H N_s} \tag{1}$$

Here $\bar{F}_s$ is the mean factor of safety computed using mean values of S_u and γ (μ_{su} and μ_γ), slope height H and slope stability number N_s. The value of $\bar{F}_s$ from Eq. (1) together with coefficients of variation of undrained shear strength (COV_{su}) and unit weight (COV_γ) were used to calculate probability of failure, P_f as follows [22]:

$$P_f = p[F_s < 1] = \Phi\left\{ \frac{\ln\left(\sqrt{\frac{1+\text{COV}_{su}^2}{1+\text{COV}_\gamma^2}}/\bar{F}_s\right)}{\sqrt{\ln\left[\left(1+\text{COV}_{su}^2\right)\left(1+\text{COV}_\gamma^2\right)\right]}} \right\} \tag{2}$$

where Φ is the cumulative standard normal distribution function. Fig. 1a and b present two sets of results. The first considers the unit weight γ to be deterministic (no variability) and hence COV_γ is equal to zero. For the second set, both undrained shear strength S_u and unit weight γ are considered as uncorrelated lognormal distributed random variables. Therefore the coefficient of variation of factor of safety is due to the variability in random variables S_u and γ, and is calculated as:

$$\text{COV}_{\bar{F}s} = \sqrt{\text{COV}_{su}^2 + \text{COV}_\gamma^2} \tag{3}$$

The dashed curves in Fig. 1a and b are the closed-form solutions using Eq. (2). The data show that as the mean factor of safety increases for any constant level of variability in random variables, the probability of failure decreases, which is expected. Unreasonably large COV values appear in these figures. They were purposely used to test the accuracy of the FEM numerical code over a wide range of input values.

Fig. 2 shows the simple slope geometry used in the simulations. In order to compare unreinforced slope FEM outcomes with pLEM results, two different groups of probabilistic analysis were conducted. In the first group the undrained shear strength S_u was the only random variable, while in the second group both the undrained shear strength S_u and the unit weight γ of the soil were treated as random variables. The mean value of the unit weight was 20 kN/m^3. All random variables are assumed to have lognormal distributions. For deterministic stability analysis for the ultimate failure limit state, the choice of Young's Modulus (E) and Poisson's ratio (v) has little influence on stability analysis outcomes [20], hence parameters E and v were taken as 100 MPa and 0.3, respectively. The mean value of S_u was varied from 30 to 60 kPa with an increment of 5 kPa. For the first group of analyses the undrained shear strength S_u was the only random variable and it was determined that 1000 Monte Carlo simulations were sufficient to give a consistent estimate of probability of failure using FEM. However, for the second group with two random variables, 2000 Monte Carlo simulations were needed to obtain a consistent result.

The simulation results for the first and second groups of simulations are plotted as symbols in Fig. 1a and b, respectively. The pLEM results based on the closed-form solution (Eq. (2)) and FEM outcomes can be seen to agree very well.

2.2.2. Cohesive–frictional (c–ϕ) soil slopes

Javankhoshdel and Bathurst [22] also developed probabilistic slope stability charts for simple unreinforced cohesive–frictional (c–ϕ) soil slopes using pLEM (LEM with Monte Carlo simulation). In these charts both cohesion c and friction angle ϕ were considered as random variables having lognormal distributions.