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Research Paper

Probabilistic reliability analysis of multiple slopes with genetic algorithms

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ABSTRACT

Uncertainty estimation and consideration in engineering is an important practice to design reliable structures especially in geotechnics since the level of control with regards to the material parameters is relatively low. The definition of reliability indices to approximate the probability of failure allows for a better assessment of stability with fewer computations than using alternative methods. Nonetheless, yet an optimisation problem needs to be solved. In this work, a genetic algorithm is developed to solve this optimisation problem considering the limit equilibrium method to search for multiple critical failures. Study cases are presented to illustrate the capabilities of the method.

1. Introduction

Slope stability is an important and challenging problem in geotechnical engineering because of uncertain material data and the potential existence of more than one failure surface. Determining the stability of slopes simply by the so-called factor of safety (FOS) is not always the best approach. In fact, the rational method for considering uncertainties in engineering problems such as multiple slope stability is via a probabilistic reliability assessment. Furthermore, material properties vary along the slope [1] and there is no assurance that the failure surface with minimum FOS is the maximum probability of failure (P_f) [2,3]. In multiple slopes, several distinct critical failure surfaces may be located at the different positions. It is actually unknown whether a single slope failure or combination of two or more slopes failures may happen. As noted in [4,5], missing critical multiple slip surfaces may lead to unsafe design.

A number of methods for deterministic slope analyses have been proposed including the limit equilibrium method (LEM) with circular and non-circular failure surface [6], the finite element method (FEM), the finite difference method (FDM) and the limit analysis method [4]. Among them, the LEMs are the most widely adopted by practitioners [4] due to its simplicity. Its major limitation is the need for postulating a potential failure surface first.

In a probabilistic reliability analysis, random soil properties are considered and the probability of failure P_f is quantified. To accomplish this, the Monte Carlo simulation (MCS) method is usually employed. This method has been published and adopted into commercial software packages such as SVSlope, FLAC, PLAXIS and GEO5. A limitation of the MCS method is its great dependency on the number of samples and random seeds. Moreover, the resulting probability of failure corresponds to only one scenario regarding the location and shape of the failure surface. Other methods include the response surface method (RSM) [7], Kriging [8] and quasi Monte Carlo [9].

For large problems such as the Vajont slope failure [10], running MCS considering every possible failure surface is a very time consuming process. Therefore, in the structural reliability field, an alternative method using the so-called Hasofer–Lind reliability index was developed in order to approximate the probability of failure. This index (β) is quantified by solving an optimisation problem and then the probability of failure is calculated by means of $P_f = \Phi(-\beta)$ with Φ being the standard cumulative distribution function.

To circumvent some of the aforementioned problems connected to Monte Carlo simulations (e.g. computational inefficiency and single slope limitation), alternative optimisation techniques such as genetic algorithms (GA) may be applied. With regards to the computational inefficiency, the large number of Monte Carlo simulations is determined with basis on the number of parameters and accuracy required. However, these parameters and accuracy

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can vary between problems (e.g. unknown number of potential failure surfaces in multiple slopes) and hence they are difficult to be computed. Here, trial and error is used to compute the number of MC simulations for the sake of comparisons.

GAs are optimisation techniques with a good popularity in engineering and many other areas [11–14] due to their relative simplicity and, more importantly, due to the limited requirements with regards to the problem formulation – basically, an objective function and a set of constraints are the only components needed. The convenience of not requiring gradients is commonly appreciated by users of genetic algorithms. In the case of multiple potential failures, GAs have an advantage because they are able to handle multimodal problems as well. Other common methods in Engineering and also classified as meta-heuristic as GA [17] are: particle swarm optimisation [4], ant colony [16], and some nature-inspired algorithms [15], to name a few.

To determine the reliability of a slope, the reliability index β is calculated first [18,19]. Other works [20–29] also confirm the importance of reliability indices and there are several methods to determine this index including methods based on genetic algorithms [30] for the problems of bearing capacity and methods based on the finite element method [31,32] for other situations. Nonetheless, the development of an efficient solution technique considering probabilistic analysis and multiple slopes stability is still under current research. This paper therefore presents a genetic algorithm in order to develop such technique.

This paper is organised as follows. In Section 2, a version of the limit equilibrium method, the FORM reliability method, and the genetic algorithm are presented. Section 3 presents the numerical studies where a number of simple cases are analysed employing the GA and are compared against results from a commercial software (SVSlope). The Vajont landslide is then assessed with regards to its pre-failure stability and the results are compared against field observation [10] in addition to other computations. Section 4 finally draws some conclusions.

2. Methods

In this work, a simple stability analysis program is developed considering the Spencer limit equilibrium method (LEM) [33]. Nonetheless, other LE methods could be equally considered. The LEM code is then combined with a probabilistic reliability assessment method where the optimisation problem is solved using an improved genetic algorithm.

2.1. Limit equilibrium method

In Spencer's method, both forces and moment equilibrium are satisfied. The interslice forces are assumed to be parallel and the angle of their inclination λ needs to be computed using an iterative technique. The factor of safety (FOS) is calculated by considering the equilibrium of forces (Eq. (1)) and the equilibrium of momentum (Eq. (2)); the following equations are solved:

$$\sum_{i=1}^{n} Q_i = 0 \tag{1}$$

$$\sum_{i=1}^{n} Q_i \left[x_{bi}\sin(\lambda) - \left(y_{bi} + \frac{M_n}{Q_i\cos(\lambda)} \right)\cos(\lambda) \right] = 0 \tag{2}$$

$$Q_i = \frac{-F_{vi}\sin(\theta_i) - F_{hi}\cos(\theta_i) - \left[\frac{c_i'\Delta l_i}{\text{FOS}}\right] + \left[F_{vi}\cos(\theta_i) - F_{hi}\sin(\theta_i) + u_i\Delta l_i\right]\frac{\tan(\phi_i')}{\text{FOS}}}{\cos(\theta_i - \lambda) + \frac{\sin(\theta_i-\lambda)\tan(\phi_i')}{\text{FOS}}} \tag{3}$$

where Q_i is the resultant force, F_{vi} and F_{hi} are the vertical and horizontal forces, respectively, θ_i is the inclination of the failure surface,

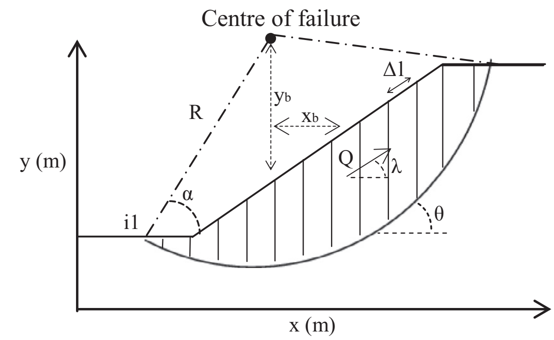

Fig. 1. Geometry and variables considered in Spencer's method.

c_i' is the effective cohesion, Δl_i is the length of slices along the failure surface, u_i is the pore water pressure, ϕ_i' is the effective frictional angle, i is the number of the slice, M_n is the moment acting on the failure surface, and x_{bi} and y_{bi} are the distances from the centre of the to the origin of the failure surface in x and y directions (Fig. 1).

2.2. Reliability analysis

The first order reliability method (FORM) is considered in this work; see, e.g. [34]. The reliability analysis starts with the quantification of the probability of failure P_f which is defined by means of

$$P_f = P(g(\boldsymbol{X}) \leqslant 0) = \int_{g(\boldsymbol{X})\leqslant 0} f(\boldsymbol{X})d\boldsymbol{X} \approx 1 - \Phi(\beta) \tag{4}$$

where $g(\boldsymbol{X})$ is the limit state function and $f(\boldsymbol{X})$ is the joint probability density function (PDF) of the random variables $\boldsymbol{X} = [X_1, X_2, \ldots, X_N]$. Therein, Φ symbolises the cumulative distribution function (CDF) of the standard normal distribution and β is the reliability index. It is worth noting that the value of P_f computed from the reliability index is only an approximation, except when the random variables are normally distributed and the limit state surface is linear. In the N-dimensional hyperspace of the basic variables, $g(\boldsymbol{X}) = 0$ is the boundary region with negative values indicating failure (Fig. 2).

The reliability index is the shortest distance from the limit state curve to the origin of the transformed space of random variables and is defined by

$$\beta = \sqrt{\boldsymbol{Y}^T\boldsymbol{C}\boldsymbol{Y}} \quad \text{with} \quad Y_i = \frac{X_i - \mu_i}{\sigma_i} \tag{5}$$

where X_i is the random variable of ith material parameter, $\boldsymbol{C}$ is the inverse of the correlation matrix between parameters and $\boldsymbol{Y}$ is a vector of normalised (transformed) variables. To illustrate the common range of values of β, Table 1 presents the reliability index values with corresponding P_f values and an auxiliary terminology regarding expected performance levels.

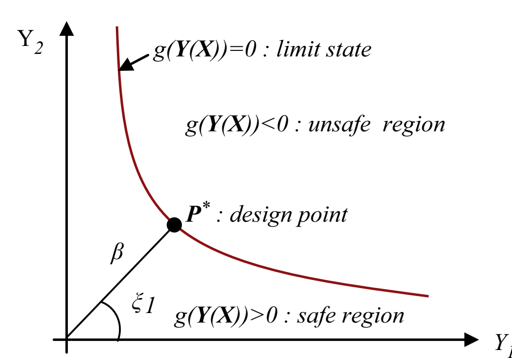

Fig. 2. Limit state and reliability index β in the normalised (mapped) space.