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Research Paper

Spectral element solution for transversely isotropic elastic multi-layered structures subjected to axisymmetric loading

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ABSTRACT

Spectral element method is proposed to analyze the mechanical response of transversely isotropic elastic multi-layered pavement structure subjected to axisymmetric loading. Based on the basic constitutive equations and modified Love's function for transversely isotropic elastic media, the governing state equation of a multi-layered transversely isotropic medium was deduced. From the equation, s spectral layer element for a single layer (i.e., a stiffness matrix) was acquired. The global stiffness matrix was obtained by assembling the interrelated layer elements based on the principle of the finite element method and the boundary conditions, and the solution for the corresponding problem was obtained by solving the algebraic equations of the global stiffness matrix. The solutions in the physical domain were acquired by means of the Fourier–Bessel superposition. The validity of the proposed solution was examined by comparing with an existing exact solution and finite element method respectively. Subsequent to solution validation, a parametric study by varying *n*-values of unbound layer was carried to investigate the influence of the transversal isotropy on pavement response. The proposed analytical solution demonstrated the ability of solving the mechanical response of transversely isotropic elastic multi-layered pavement structures subjected to axisymmetric loading.

1. Introduction

Pavement engineers have been greatly interested in the behavior of layered materials under certain loading conditions mainly due to the fact that asphalt pavements are composed of horizontal layers of materials of different types. Conventionally, although asphalt mixtures are generally considered as typical viscoelastic materials, the asphalt pavement is regarded as a layered elastic structure for mechanical response analysis with many studies already being carried out based on the well-known layered elastic theory subjected to vertical loads distributed over circular area. Burmister [1] presented the first solution for both two-layer and three-layer systems. Some computer programs have been developed based on Burmister's theory in the last decades, e.g., BISAR [2], Kenlayer [3], JULEA [4], with JULEA being further incorporated into the MEPDG [5]. Recently, several analytical approaches and solutions of multi-layered structures have laterally considered comprehensive aspects: the dynamic effect, the elastic or viscoelastic behavior of material, or the damping effect (e.g., the solutions developed by Xu [6] and Lee [7]). However, pavement materials are often assumed to be homogeneous and isotropic when stress and strain are calculated in most existing researches on flexible pavement theory.

In the last years, many researchers have found that the layer pavement materials are transversely isotropic [8–10]. Compared to a uniform elastic material model, the use of a multi-layered transversely isotropic model to describe the deformation of a layered medium is more reasonable [8,11,12]. Therefore, it is more reliable and important to consider these anisotropic properties when stress and displacement solutions for these materials are derived. The characterization and modeling of the anisotropic properties of the unbound granular aggregate layer (e.g., soils) have been widely explored in geomechanics and geotechnical engineering [13–16]. Several finite element methods (FEM) studies have investigated the effects of pavement materials' transverse anisotropy on asphalt pavement mechanical responses [17,18], however, there are few analytical studies for pavement response based on transversely isotropic theory.

In the decades, the spectral element method developed by Doyle [19] combines elegantly the exact solution of wave motions with the finite element organization of the system matrices. In this approach, the system is solved by double summation over the

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involved frequencies and the wave numbers [20], alleviating thus the inconvenience of the numerical implementation of infinite integration. In the spectral element technique, an analytical layer element is used to describe a single layer, which not only reduces the computational requirement dramatically but also demonstrates the numerical efficiency and stability due to the absence of positive exponential functions and evades the inconvenience of the numerical evaluation of contour integration between zero and infinity. Recently, the spectral elements method is utilized for the analysis of the dynamic behavior of pavement structures under the impact of the FWD load pulse comprehensively [21–23].

The main objective of this paper was to extend the spectral element method to analyze a transversely isotropic elastic medium subjected to axisymmetric static loading, and then present an alternative algebraic formulation for the development of the analytical and numerical techniques for transversely isotropic elastic pavement. As an application, the case of pavement response under the vertical loads distributed over circular area is validated. The influences of the degree of transverse anisotropy (i.e., the ratio of stiffness or modulus in horizontal and vertical directions, n-value) of the multi-layered pavement on the displacements and strains response are discussed.

2. Governing state equation

The vertical load applied to the surface is uniformly distributed over a circular area, which leads to an axially symmetric problem. The equations of the theory of elasticity for the three-dimensional problem in cylindrical coordinates as used in this study are summarized in the following.

The partial differential equations of equilibrium without body force in the cylindrical coordinate system are given by [24]:

$$\begin{cases} \dfrac{\partial \sigma_r}{\partial r} + \dfrac{\partial \tau_{rz}}{\partial z} + \dfrac{\sigma_r - \sigma_\theta}{r} = 0 \\ \dfrac{\partial \sigma_z}{\partial z} + \dfrac{\partial \tau_{rz}}{\partial r} + \dfrac{\tau_{rz}}{r} = 0 \end{cases} \tag{1}$$

where σ and τ are the normal and shear stress respectively, and r, θ, and z are the radial, circumferential, and axial coordinate respectively (Figs. 1 and 2).

The relationship of strain and displacement are [24]:

$$\varepsilon_r = \frac{\partial u_r}{\partial r}, \quad \varepsilon_\theta = \frac{u_r}{r}, \quad \varepsilon_z = \frac{\partial u_z}{\partial z}, \quad \gamma_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \tag{2}$$

where u is displacement.

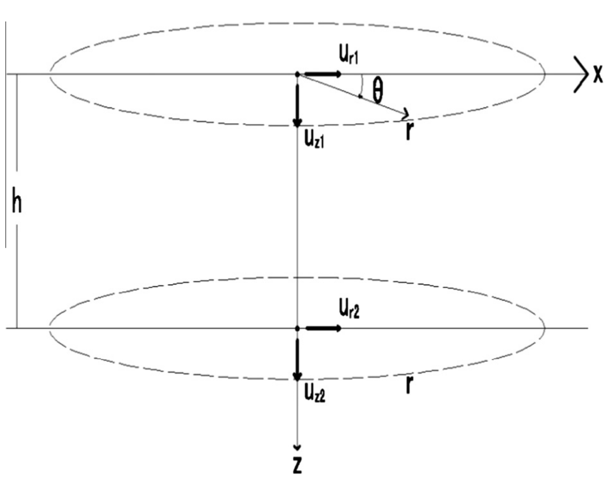

Fig. 1. 2-noded axi-symmetric layer element.

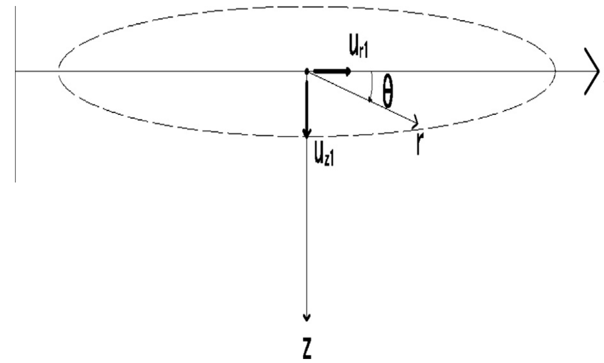

Fig. 2. 1-noded half-space element.

The relationship of strain and stress for transversely isotropic elastic medium can be represented as [24]:

$$\begin{Bmatrix} \sigma_r(r,z) \\ \sigma_\theta(r,z) \\ \sigma_z(r,z) \\ \tau_{rz}(r,z) \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 \\ c_{21} & c_{11} & c_{13} & 0 \\ c_{13} & c_{31} & c_{33} & 0 \\ 0 & 0 & 0 & c_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_r(r,z) \\ \varepsilon_\theta(r,z) \\ \varepsilon_z(r,z) \\ \gamma_{rz}(r,z) \end{Bmatrix} \tag{3}$$

where c_{ij} are elastic coefficients, and given by

$$\begin{cases} c_1 = \kappa n(1 - n\mu_v^2), \quad c_{12} = \kappa n(\mu_h + n\mu_v^2) \\ c_{13} = \kappa n\mu_v(\mu_h + 1), \quad c_{33} = \kappa(1 - \mu_h^2) \\ c_{44} = G = E_v/2(1 + \mu_v) \end{cases} \tag{4}$$

Here ε_θ, ε_z and γ_{rz} are the components of strain. E_v, E_h and G are Young's modulus in the vertical direction, Young's modulus in the horizontal direction, shear modulus in planes normal to the plane of transverse isotropy, respectively, μ_h and μ_v are Poisson's ratios in the horizontal direction and in the vertical direction, respectively. Where the constants n and κ are defined as

$$n = \frac{E_h}{E_v}, \quad \kappa = \frac{E_v}{(1 + \mu_h)(1 - \mu_h - 2\kappa\mu_v^2)} \tag{5}$$

By introducing Eq. (2) into Eq. (4), the above stress equation:

$$\begin{cases} \sigma_r(r,z) = c_{11}\dfrac{\partial u_r}{\partial r} + c_{12}\dfrac{u_r}{r} + c_{13}\dfrac{\partial u_z}{\partial z} \\ \sigma_\theta(r,z) = c_{12}\dfrac{\partial u_r}{\partial r} + c_{11}\dfrac{u_r}{r} + c_{13}\dfrac{\partial u_z}{\partial z} \\ \sigma_z(r,z) = c_{13}\dfrac{\partial u_r}{\partial r} + c_{13}\dfrac{u_r}{r} + c_{33}\dfrac{\partial u_z}{\partial z} \\ \tau_{rz}(r,z) = c_{44}\left(\dfrac{\partial u_r}{\partial r} + \dfrac{\partial u_z}{\partial z}\right) \end{cases} \tag{6}$$

Substituting Eqs. (2) and (4) into Eq. (1), leads to the following equations:

$$\begin{cases} \left(\dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} - \dfrac{1}{r^2} + \dfrac{c_{44}}{c_{11}}\dfrac{\partial^2}{\partial z^2}\right)u_r + \dfrac{(c_{13} + c_{44})}{c_{11}}\dfrac{\partial^2}{\partial r \partial z}u_z = 0 \\ \dfrac{c_{13} + c_{44}}{c_{44}}\left(\dfrac{1}{r} + \dfrac{\partial}{\partial r}\right)\dfrac{\partial u_r}{\partial z} + \left(\dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial r^2} + \dfrac{c_{33}}{c_{44}}\dfrac{\partial^2}{\partial z^2}\right)u_z = 0 \end{cases} \tag{7}$$

The deflection in Eq. (7) can be written as the following in terms of the modified Love's displacement function [25]

$$\begin{cases} u_r(r,z) = -\dfrac{\partial^2 \varphi(r,z)}{\partial r \partial z} \\ u_z(r,z) = \left[a_1\left(\dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r}\right) + b_1\dfrac{\partial^2}{\partial z^2}\right]\varphi(r,z) \end{cases} \tag{8}$$