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Research Paper

A semi-analytical solution for the transient response of one-dimensional unsaturated single-layer poroviscoelastic media

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ABSTRACT

This paper develops a semi-analytical solution for the transient response of an unsaturated single-layer poroviscoelastic medium with two immiscible fluids by using the Laplace transformation and the state-space method. Using the elastic–viscoelastic correspondence principle, we first introduce the Kelvin–Voigt model into Zienkiewicz's unsaturated poroelastic model. The vibrational response for unsaturated porous material can be obtained by combining these two models and assuming that the wetting and non-wetting fluids are compressible, the solid skeleton and solid particles are viscoelastic, and the inertial and mechanical couplings are taken into account. The Laplace transformation and state-space method are used to solve the basic equations with the associated initial and boundary conditions, and the analytical solution in the Laplace domain is developed. To evaluate the responses in the time domain, Durbin's numerical inverse Laplace transform method is used to obtain the semi-analytical solution. There are three compressional waves in porous media with two immiscible fluids. Moreover, to observe the three compressional waves clearly, we assume the two immiscible fluids are water and oil. Finally, several examples are provided to show the validity of the semi-analytical solution and to assess the influences of the viscosity coefficients and dynamic permeability coefficients on the behavior of the three compressional waves.

1. Introduction

The theory of porous media has been widely applied in many fields, ranging from geotechnical engineering to bioengineering [1]. A porous medium is considered saturated if it is filled with a single type of fluid and considered unsaturated if it is filled with multiple fluids [2]. In most cases, there are two fluids coexisting within the pore space. For example, in petroleum engineering applications, oil and water may exist in the pore space, while in geotechnical engineering applications, air and water may exist in the pore space.

Since Biot [3] established the theory of porous media, many significant achievements have been made on the wave motion in porous media [4,5]. However, due to the complexity of the inertial, viscosity, and mechanical couplings in porous media, scholars usually use numerical methods involving the discretization of both the spatial and temporal domains to solve transient response problems. Even for the one-dimensional (1D) problems, only a few analytical solutions are provided in the literature.

Several analytical solutions have been published for the 1D transient response of saturated and unsaturated poroelastic media. Garg et al. [6], Simon et al. [7], and de Boer et al. [8] studied the problem of a saturated semi-infinite porous medium; they derived analytical solutions in a time domain in several special cases by using the Laplace transform method. For the saturated single-layer problem, Schanz and Cheng [9] obtained an analytical solution in the Laplace domain. In addition, Gajo and Mongiovi [10] and Shan et al. [11,12] derived several analytical solutions by adopting the method of separation of variables. For unsaturated single-layer problem, Li and Schanz [13] gave an analytical solution in the Laplace domain, and Shan et al. [14] obtained analytical solutions for three types of boundary conditions through the method of separation of variables.

A poroviscoelastic model could consider the viscoelastic shear behavior of the solid skeleton and damping mechanism of the solid particles [15,16]. Therefore, the poroviscoelastic model has a much wider application than the poroelastic model. Moreover, the poroviscoelastic model can reduce to the poroelastic model.

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Studies conducted on the wave motion in poroviscoelastic media are few compared with the studies on poroelastic media. The Laplace transform method is the major method in dealing with viscoelastic problems [17,18]. For the 1D problem, Schanz and Cheng [16] developed the governing equation for saturated poroviscoelastic media by introducing the Kelvin–Voigt model and obtained several semi-analytical solutions. Pisanó and Pastor [19] conducted a numerical simulation of wave propagation by using an improved fractional step, the Taylor–Galerkin finite element algorithm. Shan et al. [20] derived an analytical solution in a time domain for a typical boundary condition by applying the finite Fourier transform method. However, no numerical solution or analytical solution has been published in the literature for the transient response of 1D unsaturated poroviscoelastic media.

Analytical solutions in a time-domain and frequency-domain can accurately illustrate the wave motion in porous media and can be used to verify the numerical calculations. Clear insight into the transient response is an essential part in the investigation of wave motion in porous media. Moreover, studying transient responses of saturated and unsaturated single layer soil is an essential part of the research on "the seismic response of the engineering site" [21]. Therefore, analytical solutions in the time-domain and frequency domain for the transient response of saturated and unsaturated porous media are of great theoretical and practical importance, even for one-dimensional problems.

The objective of this study is to develop a semi-analytical solution in the time domain for a single-layer unsaturated poroviscoelastic medium subjected to a typical boundary condition. The article is organized as follows. In Section 2, the basic equations describing unsaturated poroviscoelastic media are first given; then, the associated initial and boundary conditions and the solution procedure of the semi-analytical solution are presented in detail in Sections 3 and 4. In the fifth section, several numerical examples are provided to verify the semi-analytical solution's correctness and to discuss the features of the three compressional waves with different viscoelastic coefficients and permeability coefficients. There are three compressional waves in porous media with two immiscible fluids. Moreover, to observe the three compressional waves clearly, we assume the two immiscible fluids are water and oil. Then, we end with a summary of the work and some concluding remarks.

2. The basic equations governing the dynamic response of the poroviscoelastic media

To establish the model for unsaturated porous media, a series of assumptions should be proposed first. Table 1 illustrates the assumptions for the unsaturated poroelastic and poroviscoelastic models, which are based on *Biot's theory*. The solid skeleton and solid particles are assumed to be viscoelastic in the poroviscoelastic model. By using the elastic–viscoelastic correspondence principle, we can obtain the basic equations for the 1D unsaturated poroviscoelastic media.

Table 1
Assumptions.

Zienkiewicz et al. [23] (poroelastic)	This paper (poroviscoelastic)
(a) The solid skeleton is a *poroelastic* medium	(a) The solid skeleton is a *poroviscoelastic* medium
(b) The solid particles are *elastic*	(b) The solid particles are *viscoelastic*
(c) Two immiscible fluids coexist within the porous space	(c) Two immiscible fluids coexist within the porous space
(d) The fluids are compressible, and the relative motions of the fluids in the pores are of the Poiseuille type	(d) The fluids are compressible, and the relative motions of the fluids in the pores are of the Poiseuille type
(e) The porous medium is isotropic, and the pores are interconnected	(e) The porous medium is isotropic, and the pores are interconnected
(f) The pore size is much less than the wavelength	(f) The pore size is much less than the wavelength
(g) The influence of temperature is not considered	(g) The influence of temperature is not considered

2.1. Basic equations for the poroelastic media

Assume that the unsaturated porous media is anchored in a Cartesian coordinate system, and the compressional waves propagate along the z direction. The basic equations for the 1D problem can be written as follows [22,23]:

The momentum balance equations for the porous media:

$$\sigma_{zz,z} - \rho U_{z,tt} - \rho_w W_{z,tt} - \rho_g V_{z,tt} = 0 \tag{1a}$$

The momentum balance equations for the two fluids:

$$\begin{aligned} &P_{w,z} + \rho_w U_{z,tt} + \rho_w W_{z,tt}/(nS_w) + W_{z,t}/k_w = 0 \\ &P_{n,z} + \rho_n U_{z,tt} + \rho_n V_{z,tt}/(nS_n) + V_{z,t}/k_n = 0 \end{aligned} \tag{1b}$$

The flow conservation equations for the two fluids:

$$\begin{aligned} &Q_w W_{z,z} + \alpha Q_w S_w U_{z,z} + S_w P_w = 0 \\ &Q_n V_{z,z} + \alpha Q_n S_n U_{z,z} + S_n P_n = 0 \end{aligned} \tag{1c}$$

The constitutive equation of the solid skeleton:

$$\sigma_{zz} + \alpha(S_w P_w + S_n P_n) = (\lambda + 2\mu) U_{z,z} \tag{1d}$$

where t and z represent time and depth, respectively. Subscripts w and n denote the wetting and non-wetting fluids, respectively. α, Q_w, and Q_n are parameters that describe the compressibility of the unsaturated porous media, and

$$\alpha = 1 - \frac{K_b}{K_s}, \quad \frac{1}{Q_w} = \frac{(\alpha - n)}{K_s} + \frac{n}{K_w}, \quad \frac{1}{Q_n} = \frac{(\alpha - n)}{K_s} + \frac{n}{K_n} \tag{2}$$

Table 2 shows other parameters.

The dynamic permeability is a useful concept that is important for analyzing seismic data as a function of frequency. The usual Darcy permeability is exactly the low-frequency limit of the dynamic permeability [24]. As the model focuses on low-frequency problems, the conventional hydraulic conductivity values ($\hat{k}_w$ and $\hat{k}_n$) and the dynamic permeability coefficients are related by $k_w = \hat{k}_w/\rho_w g$ and $k_n = \hat{k}_n/\rho_n g$, where g is the gravitational acceleration. The units of the dynamic permeability coefficients k_w and k_n are $m^4/(Ns)$.

Introducing the dimensionless variables of stresses, displacements, depth, and time:

Table 2
Parameters.

σ_{zz}	Total stress. Tension is positive
P_w, P_n	Wetting fluid pressure and non-wetting fluid pressures. Pressure is positive
U_z	Solid displacement
W_z, V_z	Relative displacements of the wetting and non-wetting fluids to the solid
K_b, K_s, K_w, K_n	Bulk moduli of the solid skeleton, solid grains, wetting fluid, and non-wetting fluid
G	Shear modulus of the solid skeleton
k_w, k_n	Dynamic permeability coefficients of the wetting fluid and the non-wetting fluid
S_w, S_n	Saturations of the wetting and non-wetting fluids, $S_w + S_n = 1$
n	Porosity
ρ_s, ρ_w, ρ_n	Densities of the solid particles and the wetting and non-wetting fluids
ρ	Density of the unsaturated porous medium, $\rho = (1-n)\rho_s + nS_w\rho_w + nS_n\rho_n$