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Research Paper

Nonstationary flow surface theory for modeling the viscoplastic behaviors of soils

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ABSTRACT

This paper presents a three-dimensional elastic viscoplastic model that can describe the time-dependent behaviors of soft clays. The constitutive model is formulated based on the nonstationary flow surface theory and incorporates new developments, including (i) an improved definition of the nonstationary flow surface that is capable of capturing the stress–strain behaviors under different loading paths, (ii) a unique stress–strain—viscoplastic-strain-rate equation that is able to explicitly describe the nonstationary flow surface, and (iii) a final stable state concept that identifies the final equilibrium state at the end of creep and stress relaxation, which is also used to simplify the loading criteria. The consistency condition is validated for the proposed model, and the viscoplastic multipliers are calculated by solving the consistency equations. The model performance is investigated and validated via simulation of both oedometer and triaxial tests. The numerical results demonstrate that the proposed model is able to reproduce the main viscoplastic behaviors of soils, including creep, undrained creep rupture, stress relaxation, rate effect and accumulated effect.

1. Introduction

Laboratory studies and field observations have highlighted the importance of considering the viscous behavior of soils in the design of geotechnical structures (e.g., [1–4]). To quantify this viscous behavior, various mathematical formulations have been developed over the past 50 years. Such formulations include empirical equations (e.g., [1,5–7]), rheological models (e.g., [8,9]) and general stress–strain–time models (e.g., [10–17]). General str ess–strain–time models are always formulated in an incremental form so that they can be easily implemented into the finite element framework. Consequently, they have been widely used to analyze practical problems such as deep foundation pits (e.g., [3]) and tunnels (e.g., [4]).

Most general stress–strain–time models are proposed based on Perzyna's overstress theory [18] due to its simplicity. Such models can be differentiated based on two significant aspects: the definition of the overstress index F and the form of the overstress function $\phi(F)$. In most works, the overstress index is defined using the dynamic yield surface (e.g., [10–12]). This procedure induces unequal expansion in different directions along the yield surface, resulting in an overestimation of the volumetric deformation under shearing [13]. To avoid this drawback, Hinchberger and Rowe [13] proposed a projection method that defines F as the distance between the original stress state and the projection stress state on the static yield surface. This projection method increases the mathematical complexity. Two forms of the overstress function have been widely used: the exponential overstress function (e.g., [10,11,19]) and the power overstress function (e.g., [13,20]). Those functions can capture the strain rate influence on the soil strength and preconsolidation pressure; however, some cannot handle the transition from the viscoplastic stage to the inviscid stage [21]. Other researchers have argued that the overstress function can be derived from the drained creep behaviors and that this method is able to simplify the determination of the model parameters [22–24]. In particular, the model of Yin and Graham [22] considered the coupled phenomena of viscoplastic strain and viscoplastic strain rate. Recently, some authors [15,20] replaced the static yield surface with a reference yield surface and introduced the unique str ess–strain—strain-rate concept to deduce the overstress function. Such an approach avoids the difficulty of determining model parameters and is able to capture the viscoplastic behavior inside the reference yield surface. However, most Perzyna

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overstress-based models assume that the viscoplastic strain rate is only related to the current stress state and not to the stress history and stress rate, thus implying that the consistency rule is not valid. These models also lack the capacity to describe tertiary creep and creep failure. This latter deficiency can be overcome by introducing a stress-state-dependent fluidity parameter [25], the destruction effect [20] or the damage law [26].

As an alternative, Non-Stationary Flow Surface (NSFS) theory was proposed by Naghdi and Murch [27] to describe the time-dependent behavior of materials and later on developed by Olszak and Perzyna [28,29]. The NSFS theory satisfies the consistency requirement and models based on NSFS theory always achieve a higher convergence rate compared to overstress based models [30]. Despite these theoretical advantages, only some constitutive models have been proposed based on NSFS theory to simulate the viscoplastic behavior of metals [31] and soils [16,17,32,33]. Among them, the Sekiguchi model [16,31] is one of the earliest models used to simulate the viscoplastic behavior of soils. This model is able to describe the creep deformation and the creep rupture under the undrained conditions. However, it predicts an infinite deformation when the time is infinite and has a limitation in describing the transition between the inviscid and the viscous behaviors. Liingaard et al. [34] summarized the limitations of NSFS based models as follows: (i) NSFS theory is unable to describe a relaxation process or a creep process when it is initiated from a stress state inside the yield surface; (ii) NSFS theory is supposed to be able to describe a relaxation process initiated from a point on the yield surface, but no description or example has been reported; and (iii) NSFS theory assumes that if a viscoplastic deformation process is first triggered, the creep process will continue to occur even though the stress state is inside the yield surface.

The main purpose of this paper is to formulate an elastic viscoplastic model based on NSFS theory, which can overcome the previously listed limitations. First, fundamental concepts of NSFS theory in addition to the clarification of the time effect are presented. The unique stress–strain—viscoplastic-strain-rate concept is then used to describe the flow yield surface, and a stable state concept is used to simplify the loading criteria. Second, a mathematical formulation is developed in the framework of the ACMEG model. The family of ACMEG models is able to capture thermal effects [35], soil structural effects [36], chemical effects [37], and non-isothermal unsaturated conditions [38] and is extended to simulate the viscoplastic behaviors in this paper. Finally, the performance of the proposed model is illustrated by simulating various tests, including constant strain rate of compression test, constant strain rate of undrained shear test, secondary consolidation test, undrained triaxial creep test, undrained triaxial stress relaxation test, and undrained shear test on overconsolidated soils.

2. Nonstationary flow surface theory

2.1. General theory framework

NSFS theory is a further development of the inviscid elastoplastic theory and the most outstanding extension is that the yield surface is assumed to flow with time, which is written as

$$f\left(\sigma'_{ij}, \alpha_m, \beta_n, t\right) = 0 \tag{1}$$

where σ'_{ij} donates the effective stress tensor, α_m is the vector of non-hardening parameters, β_n is the vector of time-independent hardening parameters, and t is time. The total strain increment $\Delta\varepsilon_{ij}$ is decomposed into an elastic strain increment $\Delta\varepsilon^{e}_{ij}$ and a viscoplastic strain increment $\Delta\varepsilon^{vp}_{ij}$ as

$$\Delta\varepsilon_{ij} = \Delta\varepsilon^{e}_{ij} + \Delta\varepsilon^{vp}_{ij} \tag{2}$$

The superscripts e and vp refer to the elastic component and viscoplastic component, respectively. The elastic strain increment is related to the effective stress increment $\Delta\sigma'_{kl}$ via

$$\Delta\varepsilon^{e}_{ij} = D^{-1}_{ijkl}\Delta\sigma'_{kl} \tag{3}$$

where D_{ijkl} is the elastic tensor. The viscoplastic strain increment is calculated according to the flow rule

$$\Delta\varepsilon^{vp}_{ij} = \lambda\frac{\partial g}{\partial\sigma'_{ij}} \tag{4}$$

where λ is a non-negative viscoplastic multiplier and g is the viscoplastic potential. Experimental test results have demonstrated that the viscoplastic strain evolves in an identical manner during both the creep process and the loading process [17,39], implying that the viscoplastic potential retains the same form regardless of whether the loading continues. The multiplier λ can be determined by solving the consistency equation.

Considering the viscoplastic strain as the only time-independent hardening parameter, the Eq. (1) can be rewritten as

$$f\left(\sigma'_{ij}, \alpha_m, \varepsilon^{vp}_{ij}, t\right) = 0 \tag{5}$$

And the consistency condition is derived as

$$\Delta f = \frac{\partial f}{\partial\sigma'_{ij}}\Delta\sigma'_{ij} + \frac{\partial f}{\partial\varepsilon^{vp}_{ij}}\Delta\varepsilon^{vp}_{ij} + \frac{\partial f}{\partial t}\Delta t = 0 \tag{6}$$

Combining (2), (3), (4) and (6), the multiplier can be deduced as

$$\lambda = -\frac{\frac{\partial f}{\partial\sigma'_{ij}}\Delta\sigma'_{ij} + \frac{\partial f}{\partial t}\Delta t}{\frac{\partial f}{\partial\varepsilon^{vp}_{ij}}\frac{\partial g}{\partial\sigma'_{ij}}} \quad \text{for the stress loading path} \tag{7a}$$

$$\lambda = -\frac{\frac{\partial f}{\partial\sigma'_{ij}}D_{ijkl}\Delta\varepsilon_{kl} + \frac{\partial f}{\partial t}\Delta t}{\frac{\partial f}{\partial\varepsilon^{vp}_{ij}}\frac{\partial g}{\partial\sigma'_{ij}} - \frac{\partial f}{\partial\sigma'_{ij}}D_{ijkl}\frac{\partial g}{\partial\sigma'_{kl}}} \quad \text{for the strain loading path} \tag{7b}$$

Comparing to the solution of the classical plastic multiplier, Eq. (7) include an additional term: $(\partial f/\partial t)\Delta t$. This extra term implies that viscoplastic strain occurs even under the constant strain condition and the constant stress condition, which correspond to stress relaxation and creep, respectively. To clarify the contribution of this term, the stress relaxation process and the creep process are explained in detail in the following section.

A stress relaxation test is illustrated in Fig. 1a (stress path PQ). Consider a soil loaded to the stress state P at the time t_0. Then, the stress relaxation process is initiated by making the total strain constant over time (the solid line in the strain–time diagram in Fig. 1a). As time advances, the stress gradually decreases from point P to point Q. According to Eq. (3), the elastic strain is also decreasing during this process (dotted line in Fig. 1a). Because the total strain is constant over time, the viscoplastic strain must increase (dashed line in Fig. 1a) to compensate the decreasing of the elastic strain.

The corresponding stress path plotted in the stress space is shown in Fig. 1b. The yield surface shrinks and passes through the point Q at the time t_1 with decreasing stress. The movement of the yield surface is dependent on the evolution of the hardening parameter ε^{vp}_{ij} and time t. The increment of the viscoplastic strain ($\|\Delta\varepsilon^{vp}_{ij}\|$ in Fig. 1a) results in an expansion of the initial yield surface to the dashed locus (Fig. 1b) if neglecting the time effect. Because the final yield surface must pass through point Q, the time t makes the yield surface shrink from the dashed locus to the final locus passing through point Q. In the other words, time softens the yield surface.