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Relationship between small and large strain solutions for general cavity expansion problems in elasto-plastic soils

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ABSTRACT

This paper presents a closed-form relationship between small and finite strain cavity expansion solutions. Its derivation is based on the non-linearly elastic–perfectly plastic cylindrical (or spherical) problem considering a general Mohr's criterion and constant plastic dilatancy. It is shown, however, that it is sufficiently accurate for general expansion problems not obeying plane-strain rotationally (or spherically) symmetric conditions and involving strain-hardening/softening constitutive behaviour. Therefore, this relationship quantifies the error stemming from the computational assumption of small deformations and provides a simple and efficient way of accounting for geometric non-linearity based entirely on conventional computational methods: 'self-correction' of small strain analyses results.

1. Introduction

Cavity expansion theory has been widely studied in the literature. A summary of numerous analytical solutions as well as of the relevant geotechnical applications has been presented by Yu [1] and will not be repeated here. The present paper focuses on the influence of the deformation formulation considered in the analysis of general cavity expansion problems.

It is well-known that solutions based on small strain theory predict a continuously increasing cavity pressure (see for example the dashed curve in Fig. 1 for the idealised case of isotropic cylindrical cavity expansion in a Mohr–Coulomb material), while cavity expansion solutions based on finite strain theory predict a limit value at large expansions (see for example the solid curve in Fig. 1). Small strain theory therefore leads to predictions of stiffer system behaviour as it is unable to reproduce an ultimate state (*e.g.* Baguelin et al. [2]; Carter et al. [3]; Soulié et al. [4]; Yu & Houlsby [5]).

This paper presents a novel closed-form relationship between small and finite strain elasto-plastic cavity expansion solutions (Section 2). The derivation of this relationship is based on several simplifying assumptions (Section 3), but it is shown by means of modelling an elliptical and an ellipsoidal cavity as well as a finite length self-boring pressuremeter test in modified Cam-clay soils under drained and undrained conditions that it is sufficiently accurate for general elasto-plastic expansion problems not obeying plane strain axial (or spherical) symmetry, with rotating principal directions (Section 4).

2. The relationship between small and large strain solutions

It will be proven in the following that the normalized wall displacement $U_{a,ls}$ (= $u_{a,ls}/a_0$, where a_0 denotes the initial cavity radius; Fig. 2a) obtained from a large strain analysis of cavity expansion in elasto-plastic soils satisfying a general Mohr's failure criterion and exhibiting constant plastic dilatancy can be expressed with respect to the normalized wall displacement $U_{a,ss}$ (= $u_{a,ss}/a_0$; Fig. 2b) obtained from a small strain analysis (and corresponding to the same cavity pressure) through the following hyperbolic function:

$$U_{a,ls} = h(U_{a,ss}) = \frac{1}{\left[1-(\zeta/\kappa_\psi+1)U_{a,ss}\right]^{\frac{1}{\zeta/\kappa_\psi+1}}} - 1, \qquad (1)$$

where the variable ζ indicates the cavity type ($\zeta = 1$, cylindrical; $\zeta = 2$, spherical) and κ_ψ is the dilation constant, expressed in terms of the dilation angle ψ as $\kappa_\psi = (1+\sin\psi)/(1-\sin\psi)$. The large strain wall displacement tends to infinity as the small strain solution approaches the critical value

$$U_{a,\lim} = \frac{1}{\zeta/\kappa_\psi+1}, \qquad (2)$$

which implies that the limit pressure $\sigma_{a,\lim}$, that is, the pressure required to create a cavity or to enlarge a cavity of finite radius indefinitely, corresponds to the cavity pressure obtained by means

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Nomenclature

a, a_0	current and initial cavity radius
b_1, b_2	major- and minor-axis radius of an ellipse/ellipsoid
c	cohesion
E	Young's modulus
F	indefinite integral of the function $1/[\zeta\,(\sigma_r - f\,(\sigma_r))]$
f	arbitrary function of the major principal stress
h	hyperbolic function
K_0	earth pressure coefficient at rest
M	slope of the critical state line
OCR	one-dimensional over-consolidation ratio
R	isotropic over-consolidation ratio
r, r_0	current and initial radius of a material point
U_a	normalized cavity wall displacement (= u_a/a_0)
$U_{a,\lim}$	critical (small strain) normalized wall displacement
u	radial displacement of a material point
u_1, u_2	radial displacement at the major and at the minor axis of an ellipse/ellipsoid
u_a	radial displacement at the cavity wall
u_ρ	radial displacement at the elasto-plastic boundary
ε_r, ε_t	radial and tangential strain
ε_r^{pl}, ε_t^{pl}	radial and tangential plastic strain
$\varepsilon_{t\rho}$	tangential strain at the elasto-plastic boundary
ζ	variable indicating the type of cavity ($\zeta = 1$, cylindrical; $\zeta = 2$, spherical)
κ_ψ	function of dilation angle
κ, λ	slope of the swelling and of the normal consolidation line in the semi-logarithmic compression plane
ν	Poisson's ratio
ρ, ρ_0	radius of the plastic zone in the current and in the undeformed state
σ_0, σ'_0	initial isotropic total and effective stress
σ_{h0}, σ_{v0}	initial horizontal and vertical stress
σ_a	cavity pressure
$\sigma_{a,\lim}$	limit cavity pressure
σ_r, σ_t	radial and tangential Cauchy stress
σ_ρ	radial stress at the elasto-plastic boundary
υ_0	initial specific volume
φ, ψ	friction and dilation angle

Subscripts

ss	small strain value
ls	large strain value

of a small strain analysis at $U_{a,\lim}$ (see for example the circular markers in Fig. 1). Setting $\kappa_\psi \to \infty$, which corresponds to the limit case of an infinitely dilatant material ($\psi = 90°$), leads to the simplified relationship

$$U_{a,ls} = h(U_{a,ss}) = \frac{U_{a,ss}}{1 - U_{a,ss}}. \tag{3}$$

Figs. 3 and 4 plot Eq. (1) for cylindrical and spherical cavity expansion, respectively, considering several dilation angles. The identity line is included in order to illustrate the small strain solution error, which, as can be seen, is practically negligible up to expansions of 10%, but increases significantly for greater values.

The hyperbolic function h satisfies all of the required conditions for theoretical consistency: (i) $h\,(0) = 0$ and $h'\,(0) = 1$ (*i.e.* coincides with the small strain solution as $U_{a,ss}$ tends to zero); (ii) $h\,(U_{a,\lim}) = h'\,(U_{a,\lim}) = \infty$ (*i.e.* approaches infinity asymptotically as $U_{a,ss}$ tends to $U_{a,\lim}$); (iii) $h'\,(U_{a,ss}) > 0$ and $h''\,(U_{a,ss}) > 0$ (*i.e.* is a monotone increasing and convex function).

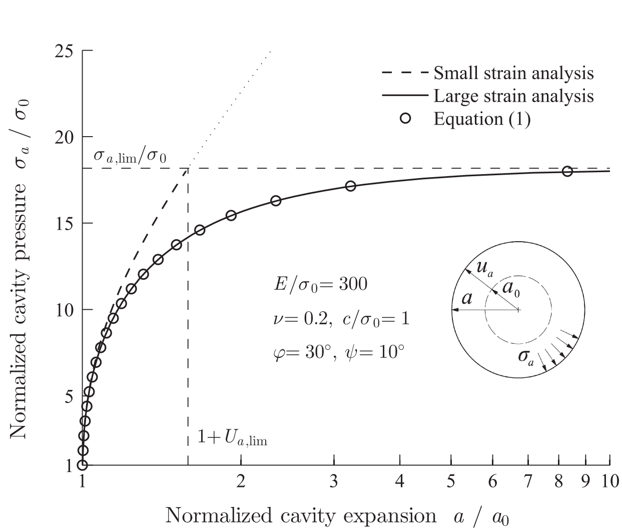

Fig. 1. Small strain, large strain and corrected small strain (Eq. (1)) cylindrical cavity expansion curves based on the Mohr–Coulomb yield criterion.

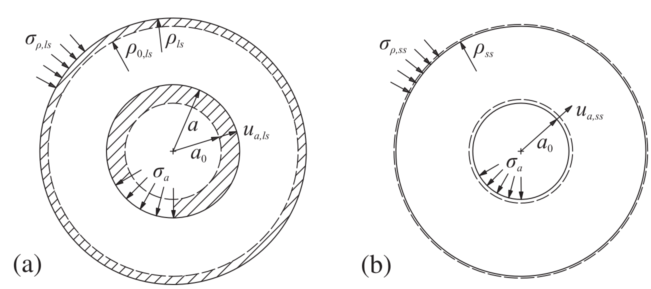

Fig. 2. Setup of the cylindrical (or spherical) cavity excavation problem for (a) large strain formulation and (b) small strain formulation.

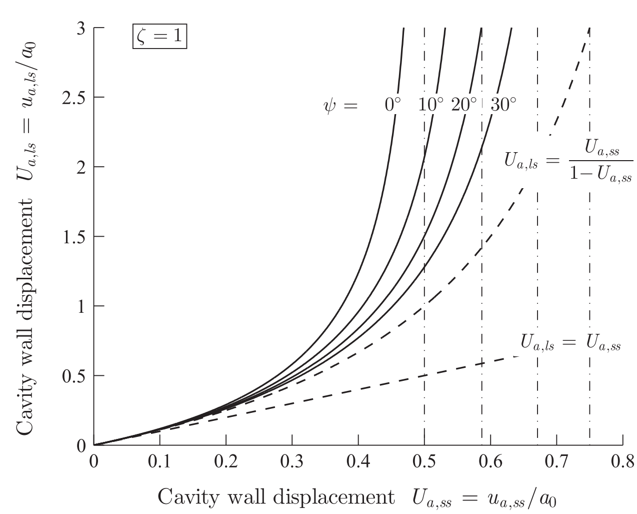

Fig. 3. Normalized cavity wall displacement from large strain analysis as a function of the normalized cavity wall displacement from small strain analysis of cylindrical cavity expansion, considering several dilation angles (Eq. (1)).