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Research Paper

An elastic–plastic model for porous rocks with two populations of voids

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A R T I C L E I N F O

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A B S T R A C T

In the present work, a new elastic–plastic model is proposed for a class of porous rock-like materials with two populations of pores at different scales. This model is based on the closed-form plastic criterion which was established from a nonlinear homogenization procedure in our previous work (Shen et al., 2014) . This criterion explicitly takes into account the effects of two populations of voids, respectively distributed at the microscopic and mesoscopic scales. In order to consider the plastic compressibility and pressure dependency, the solid phase at the microscopic scale is assumed to obey to a Drucker–Prager criterion. The constitutive model is completed by a non-associated plastic flow rule and an isotropic hardening law, which are defined in a phenomenological way. The proposed model is applied to describe the macroscopic mechanical behavior of the Lixhe chalk with different confining pressures. Comparisons between numerical results and experimental data are presented for the verification of the proposed model.

1. Introduction

Most geomaterials are porous media with complex distributions of pores. Pores of different sizes can be found at various scales. Take the example of porous chalk. According to some previous works [20,28,1], two main families of pores can be observed in these rocks, large pores corresponding to inter-particle voids between coccolithe grains and intra-particle small pores inside the coccolithe grains.

The macroscopic mechanical behavior, for instance the elastic modulus and plastic yield thresholds, are strongly affected by these two families of pores [10,17,20,4,1]. Due to the presence of pores, the mechanical behavior of chalk is very sensitive to confining pressure. Two mechanisms of plastic deformation can be identified, plastic pore collapse under hydrostatic stress and plastic shearing under deviatoric stress. A number of phenomenological constitutive models have been developed for the description of mechanical behavior of various chalks and we do not intend to give an exhaustive list of such models. These phenomenological models are able to reproduce overall mechanical responses of chalk but fail to explicitly take into account effects of pores. Empirical laws were generally proposed to reproduce variations of mechanical properties with porosity.

In order to complete and improve the macroscopic approach, important efforts have been spent to formulate micromechanics based constitutive models for porous materials. As a pioneer work, Gurson [8] proposed an analytical failure criterion based on a limit analysis approach, for porous metal materials constituted of a von Mises type solid containing a spherical void. This criterion explicitly depends on the porosity of material. Based on this reference work, a large number of extensions and improvements have been developed by various authors for different kinds of engineering materials including rock-like materials. For example, using a Drucker–Prager type pressure-sensitive criterion for the solid matrix, Jeong [11]; Guo et al. [7]; Lee and Oung [12]; Durban et al. [5]; Shen et al. [24] have formulated closed-form plastic criteria for porous materials whose solid matrix exhibits irreversible volumetric compressibility or dilation. On the other hand, Cazacu and Stewart [3]; Monchiet et al. [15] have taken into account the tension–compression asymmetry and the anisotropy of the solid matrix while Gologanu et al. [6]; Pardoen and Hutchinson [18]; Monchiet et al. [16] considered the voids shape effects. Further, the Gurson's model was adopted by Xie and Shao [30] as the cap yield locus in their two yield surfaces model. The macroscopic criterion derived by Guo et al. [7] was also applied by Lin et al. [13] to describe plastic deformation of porous rocks.

However, all micro-mechanical models mentioned above consider only one population of pores. In practice, in many rock-like materials, there exist different families of pores with significantly different sizes at different material scales. As a consequence, the effect of porosity on macroscopic behaviors of porous materials can be different at different scales, in particular when effects of pore pressure should be considered in future works. There is a need

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to develop micro-mechanical models which are able to consider multi-scale populations of pores. In the present paper, we are limited to rock-like materials with two distinct populations of pores at two different scales. To this end, Vincent et al. [29] succeed to derive a semi analytical expression of the macroscopic criterion for porous materials with two populations of pores and a von Mises solid phase at the microscopic scale. The same material has been studied in Shen et al. [23]. However, it is found that the obtained elliptic plastic yield surface does not fit well most experimental yield stress observed in laboratory tests [10,17,20,30]. Recently, a closed-form macroscopic criterion has been derived by Shen et al. [21] for double porous materials whose solid phase obeys to a Drucker–Prager criterion.

The main objective of the present study is to propose a new micromechanics based plastic model for a class of porous rock-like materials with two populations of pores and a pressure sensitive solid phase. For this purpose, a two-step nonlinear homogenization procedure will be proposed. The first step consists of determining the effective plastic criterion of porous grains with the effect of small pores by taking advantage of the previous results [14]. At the second step, a limit-analysis approach will be developed to formulate the macroscopic yield criterion of porous rocks taking account the effect of large pores.

The paper is organized as follows. The determination of the effective plastic yield criterion of porous rock is first presented in Section 2 based on the previous works of Shen et al. [21]. After introducing a non-associated plastic potential, the micromechanics based plastic model is completed in Section 3. This model is then applied to describe the mechanical behavior of the Lixhe chalk. The performance of the proposed model is finally verified through comparisons between numerical results and experimental data.

2. Macroscopic criterion of double porous materials

2.1. The problem statement

As mentioned above, a two-step homogenization procedure will be developed. For this purpose, it is needed to choose an appropriate representation of the micro-structures of double porous materials. In the present study, for the sake of simplicity, we assume that both families of pores are of spherical form and randomly distributed in a solid matrix. Therefore, the representative volume element (RVE) of the studied porous material is defined in Fig. 1. At the macroscopic scale, the studied material can be seen as an equivalent homogeneous material (see Fig. 1a). The inter-particle pores (large pores) of the double porous medium are found at the mesoscopic scale. The matrix in Fig. 1b itself is a porous medium which is composed of intra-particle pores (small pores) and the solid phase at the microscopic scale (Fig. 1c).

The two populations of spherical voids are distributed at two well separated scales. We denote $|\Omega|$ the total volume of the RVE, Ω_2 the volume of the large voids at the mesoscopic scale, Ω_1 and Ω_m are the domains occupied by the small voids and the solid phase at the microscopic scale, respectively. With these notations, the porosity at the microscopic scale (intra-particle pores) f, the one at the mesoscopic scale ϕ (inter-particle pores) and the total porosity Γ at the macroscopic scale can be expressed as:

$$f = \frac{|\Omega_1|}{|\Omega - \Omega_2|}, \quad \phi = \frac{|\Omega_2|}{|\Omega|}, \quad \Gamma = \frac{|\Omega_1| + |\Omega_2|}{|\Omega|} = f(1-\phi) + \phi \tag{1}$$

The problem to be solved here consists first in the formulation of effective plastic criterion of the porous grains with the micro porosity f in the first step of homogenization and then in the determination of the macroscopic plastic criterion of the porous material with considering the meso porosity ϕ.

Based on a hollow sphere as illustrated in Fig. 1b, a macroscopic yield criterion has been established in Shen et al. [21] for this class of porous material having two populations of pores at different scales by using a two-step homogenization procedure. By using this criterion, a micromechanical model will be developed in this work which takes into account the effects of intra-particle pores and the compressibility of the solid phase at the microscale and the influence of inter-particle pores at the mesoscale. Because the detail of derivation of the macroscopic yield criterion can be found in Shen et al. [21], here we briefly recall the main steps to better understand this criterion. The sign convention of stress and strain are: tensile stress (strain) is positive whereas compressive stress (strain) is negative.

2.2. Homogenization from microscopic to mesoscopic scale

For most geomaterials, the plastic behavior is generally affected by the mean stress and exhibits volumetric compressibility or dilatancy. In the first homogenization step from the microscopic scale to mesoscopic scale (see Fig. 1c), the plastic behavior of the solid phase is here assumed to obey a Drucker–Prager criterion:

$$\Phi^m(\tilde{\boldsymbol{\sigma}}) = \tilde{\sigma}_d + T(\tilde{\sigma}_m - h) \leq 0 \tag{2}$$

where $\tilde{\boldsymbol{\sigma}}$ denotes the local stress in the solid phase, $\tilde{\sigma}_m = \mathrm{tr}\tilde{\boldsymbol{\sigma}}/3$ the mean stress, and $\tilde{\sigma}_d = \sqrt{\tilde{\boldsymbol{\sigma}}' : \tilde{\boldsymbol{\sigma}}'}$ the equivalent deviatoric stress with $\tilde{\boldsymbol{\sigma}}' = \tilde{\boldsymbol{\sigma}} - \tilde{\sigma}_m \mathbf{1}$. The symbol "$\sim$" is used in order to make difference

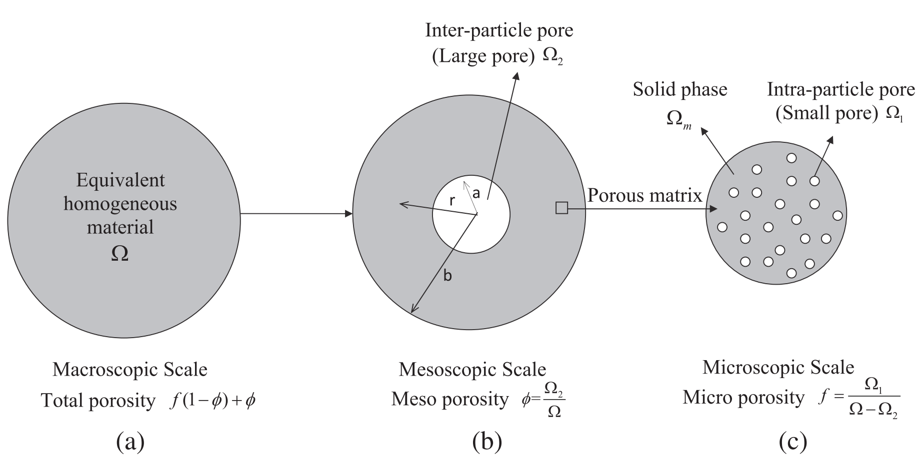

Fig. 1. The RVE of the studied double porous medium with different scales.