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Research Paper

Modeling heat and mass transfer during ground freezing subjected to high seepage velocities

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ABSTRACT

Natural or induced groundwater flow may negatively influence the performance of artificial ground freezing: high water flow velocities can prevent frozen conditions from developing. Reliable models that take into consideration hydraulic mechanisms are then needed to predict the ground freezing development. For forty years, numerous thermo-hydraulic coupled numerical models have been developed. Some of these models have been validated against experimental data but only one has been tested under high water flow velocity conditions. This paper describes a coupled thermo-hydraulic numerical model completely thermodynamically consistent and designed to simulate artificial ground freezing of a saturated and non-deformable porous medium under seepage flow conditions. On some points, less restrictive assumptions than the ones usually used in the literature are considered. As for the constant-porosity assumption, its validity is verified. The model appears to be well validated against analytical solutions and a three-dimensional ground freezing experiment under high seepage flow velocity conditions. It is used to highlight key thermo-hydraulic mechanisms associated with phase change in a porous medium.

1. Introduction

Physical processes associated with frozen ground, either natural or artificial, have been extensively studied (see for example [1,2]). Perennially or seasonally frozen ground in cold regions are of interest particularly for civil engineers (frost heave, changes in mechanical behavior) and hydrogeologists (redistribution of water). As for artificial ground freezing, it has a broad range of applications, from tunnels to landslides stabilization through shaft sinking and containment of hazardous waste. Especially in the case of artificial frozen ground, natural or induced groundwater flow may have a strong impact on the development of the frozen conditions. Conversely, phase change can affect the water velocity field due to both the difference in density between liquid water and ice and the cryo-suction process [3].

These processes explain the motivation to develop models coupling thermal and hydrogeological mechanisms in frozen ground, since the first model elaborated by [4]. Most of them have been reviewed by [5–7]. It appears that the differences in theoretical formulation of these models results from the varied backgrounds of their authors. Typically, the models use different forms of soil freezing characteristic curve, Clapeyron equation, and thermal and hydraulic conductivity relationships. A number of these models have been validated against experimental data. But, to our knowledge, only the model presented by [8] has been verified for high water flow velocity conditions, even if some authors such as [9] use their model for applications submitted to such conditions. However, high water flow velocities can delay or even prevent the freezing progress in a ground, since flowing groundwater adds heat. The effect of such conditions has then to be taken into account in an appropriate and reliable manner. As a general rule of thumb, it has been suggested that the whole space between two freeze pipes may not freeze if the water velocity is greater than 1–2 m/d [2]. Therefore, highly permeable materials combined with high hydraulic gradients are of particular concern for ground freezing. Two examples can be cited. The first one deals with a tunnel beneath the Limmat river in Zurich which experienced significant delays due to the effect of the seepage flow on the closure of the frozen body. As for the second, it concerns the Fürth subway in Germany where the effect of the groundwater flow, perpendicular to the tunnel, could not be neglected for the design of the freezing system [10].

The main objective of this paper is to present a thermo-hydraulic (TH) coupled model of a saturated porous medium subject to freezing and to validate it under conditions of high seepage

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flow velocity. For certain aspects, this model considers more general assumptions than the ones commonly presented in the literature. In particular, the ice pressure is not necessarily assumed equal to the zero gauge pressure (unlike, for example, [11–13]). Indeed, after [14], in saturated zones the ice pressure is forced to be nonzero, unlike in unsaturated zones where ice can grow without resistance for expansion. Moreover, the difference in density between ice and water is not ignored (unlike, for example, [8,12]) and thus the excess of liquid pressure caused by volume expansion during freezing can be simulated. Furthermore, the model is thermodynamically consistent and, contrary to what is generally done in the literature, all the governing equations are redemonstrated so that all the assumptions required to produce the final model and what they involve are known. The Clapeyron equation is not needed to express the equilibrium relationship between temperature and pressure in frozen ground. Instead, the direct expression of the water and ice Gibbs energies is used.

In the first part of this paper, the theoretical formulation of the fully coupled model is presented from the basis of thermodynamics, along with the underlying assumptions. Then, the model is verified against two analytical solutions and the 3D experiment conducted by [15], which involves high water flow velocities. In a third part, the validity of the constant-porosity assumption is verified by investigating the influence of porosity variation on freezing evolution. Finally, the model is applied to highlight the principal TH coupled processes associated with ground freezing.

2. Formulation of the thermo-hydraulic model

2.1. Balance equations

2.1.1. Theoretical development of a general form for the macroscopic balance laws

The subject under study in this paper is a porous medium considered fully saturated by water and subjected to freezing: it is constituted by a solid skeleton including empty spaces through which one or several water phases (liquid water and ice) can circulate. In other words, it is constituted of three phases: soil particles, liquid water and solid water or ice. It is assumed that the water is totally pure, i.e. that liquid water and ice phases are mono component. In particular, the effects of solutes are not taken into consideration. The following section intends to establish the balance equations governing the thermal and hydraulic behavior of the porous medium.

Concerning the water phases, each phase is regarded as a single-component open system exchanging matter with the outside. In what follows, the greek letters $(\alpha, \ldots)$ designate the phases: $\alpha = \sigma$ for soil particles, $\alpha = \lambda$ for liquid water and $\alpha = \gamma$ for ice.

Each phase occupies a domain Ω_α and its movement can be described in an independent way. In particular, one can define for each particle of a phase its velocity vector $\vec{v}_\alpha$ at each time t. Within each phase α, the balance law for a mass density $\varphi_\alpha\left(\vec{x}, t\right)$, assumed continuous and differentiable, of a global additive quantity may be written under the general local conservative form:

$$\partial_t(\rho_\alpha \varphi_\alpha) + \vec{\nabla}\cdot\left(\rho_\alpha \varphi_\alpha \vec{v}_\alpha + \vec{\Psi}_\alpha\right) = \rho_\alpha \varphi^\star_\alpha, \quad \forall(\vec{x}, t) \in \Omega_\alpha \times \mathbb{R}^+ \tag{1}$$

where ρ_α is the phase density, $\vec{v}_\alpha$ is the velocity of the medium's particles, $\varphi^\star_\alpha$ is a source mass density and $\vec{\Psi}_\alpha$ is a surface flux density. In what follows, a thermal equilibrium, instantaneously established, is assumed between the phases: all phases have the same temperature T.

The balance laws are gathered in Table 1. The first three are, from top to bottom: conservation of mass, balance of momentum and balance of total energy $e = \kappa + u$, where $\underline{\sigma}$ is the symmetric Cauchy stress tensor, $\vec{\psi}$ is the flux density of the rate of heat received by the domain by conduction through its boundary, and r is a volumetric density defining a rate of heat supplied to the domain by the outside. The balance laws of kinetic energy $\kappa = \vec{v}\cdot\vec{v}/2$ and internal energy u were added to Table 1. These last two laws do not constitute additional laws since they result directly from the principal balance laws. In addition, for a fluid phase, i.e. for liquid water and ice here (since it is assumed that ice is surrounded by liquid water), $\underline{\sigma}$ can be broken down between the pressure and the viscous stresses: $\underline{\sigma} = -p\underline{1} + \underline{\zeta}$. The balance law of enthalpy $h = u + p/\rho$ may then be written as:

$$\partial_t(\rho_\alpha h_\alpha) + \vec{\nabla}\cdot\left(\rho_\alpha h_\alpha \vec{v}_\alpha + \vec{\psi}_\alpha\right) = r_\alpha + \underline{\zeta}_\alpha : \underline{\underline{\nabla}}\vec{v}_\alpha + \partial_t p_\alpha + \vec{v}_\alpha\cdot\vec{\nabla}p_\alpha \tag{2}$$

Eq. (1) must be supplemented by the jump relations associated with the balance laws at each interface $\alpha\beta$ between phase α and β:

$$\left[\rho_\alpha\varphi_\alpha(\vec{v}_\alpha - \vec{v}_{\alpha\beta}) + \vec{\Psi}_\alpha\right]\cdot\vec{n}_\alpha + \left[\rho_\beta\varphi_\beta(\vec{v}_\beta - \vec{v}_{\alpha\beta}) + \vec{\Psi}_\beta\right]\cdot\vec{n}_\beta = -\zeta_{\alpha\beta}, \quad \forall(\vec{x}, t) \in \Sigma_{\alpha\beta}\times\mathbb{R}^+ \tag{3}$$

where $\Sigma_{\alpha\beta}$ is the surface describing the interface $\alpha\beta$, $\vec{v}_{\alpha\beta}$ is the interface's velocity, $\vec{n}_\alpha$ is the exterior normal unit vector to phase α with $\vec{n}_\alpha = -\vec{n}_\beta$, and $\zeta_{\alpha\beta}$ is the thermodynamic property associated with the interface. If the surface has no thermodynamic properties, $\zeta_{\alpha\beta} = 0$.

The approach adopted by Eqs. (1) and (3) can be labelled as 'microscopic', which is almost impossible to apply to practical engineering applications that need 'macroscopic' equations. To obtain them, the volume averaging method [16] is generally adopted over a Representative Elementary Volume (REV) of the porous medium. This method, applied to each phase, assumes that each macroscopic point $\vec{x}$ of the porous medium can be seen as the superposition of several phases α considered as continuous media. It can be noted that the REV should be defined as very small compared to the characteristic dimension of the problem, which is the freeze pipe diameter in our case, in the order of 10 cm. For our problem, it is assumed that the laws that are determined in the laboratory for a big REV (a few centimeters) still apply. But to complement the experimental measurements of the medium properties made at the laboratory, the numerical results also need to be fit to *in situ* measurements.

For the volume averaging method, several definitions are introduced. The volume fraction n_α of phase α is defined as the ratio of the volume of the REV part occupied by phase α to the total REV volume: $n_\alpha(\vec{x}, t) = \delta\mathcal{V}_\alpha/\delta\mathcal{V}$. For a quantity $\varphi(\vec{x}, t)$ defined over the domain $\Omega(\vec{x})$, the volumetric average is defined as:

Table 1
Balance equations.

$\partial_t(\rho_\alpha \varphi_\alpha) + \vec{\nabla}\cdot\left(\rho_\alpha \varphi_\alpha \vec{v}_\alpha + \vec{\Psi}_\alpha\right) = \rho_\alpha \varphi^\star_\alpha$

Mass density φ_α	Source term $\varphi^\star_\alpha$	Surface flux density $\vec{\Psi}_\alpha$
1	0	$\vec{0}$
$\vec{v}$	$\vec{g}$	$-\underline{\sigma}$
e	$\vec{g}\cdot\vec{v} + r/\rho$	$-\underline{\sigma}\vec{v} + \vec{\psi}$
κ	$\vec{g}\cdot\vec{v} - \left(\underline{\sigma} : \underline{\underline{\nabla}}\vec{v}\right)/\rho$	$-\underline{\sigma}\vec{v}$
u	$\left(r + \underline{\sigma} : \underline{\underline{\nabla}}\vec{v}\right)/\rho$	$\vec{\psi}$