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Research Paper

Strength prediction of dry and saturated brittle rocks by unilateral damage-friction coupling analyses

Qi-zhi Zhu *

College of Civil and Transportation Engineering, Hohai University, Nanjing, China

A R T I C L E I N F O

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A B S T R A C T

Crack growth and frictional sliding are two essential dissipative mechanisms that govern nonlinear mechanical behaviors of brittle rocks. Usually, unilateral contact as well as damage-friction coupling at cracks render great difficulties to analytical prediction of rock failure. This paper aims at deriving a new strength criterion through homogenization-based unilateral damage-friction coupling analyses. For closed frictional cracks, failure functions of the Mohr–Coulomb type and the Drucker–Prager type are obtained from a generalized friction criterion with back-stress hardening/softening. For open cracks, an elliptical failure function is achieved from a strain energy-release rate based damage criterion. Thus, the resulting strength criterion consists of two parts, whose continuity and smoothness at any crack opening-closure transition is guaranteed theoretically, independently on material properties. The basic results are then extended to take into account pore pressure effect and a simple relation between the strength envelopes for the dry and saturated cases is found. In addition, original discussions are delivered on basic features required by the damage resistance in order to capture strain hardening/softening of cracked rocks. By predicting brittle failure from constitutive equations, this work favors the development of a theoretically unified and thermodynamically consistent micro–macro model.

1. Introduction

When subjected to increasing mechanical loads, brittle rocks exhibit nonlinear quasi-brittle behaviors and generally experience successively strain hardening and damage softening, their transition corresponding to material strength. In that process, damage by microcracking as well as frictional sliding along closed cracks' surfaces are commonly considered as main dissipative mechanisms that govern inelastic deformation, damage evolution and progressive failure of cohesive geomaterials [1–3]. Theoretical studies confirmed that the above two mechanical mechanisms allow to explain quite reasonably and satisfactorily a major part of laboratory phenomena involved in brittle rocks, such as nonlinearity of mechanical responses, poromechanical coupling behaviors, induced anisotropies of the material, volumetric strain dilatancy, pressure dependency, asymmetry of strength in tension and in compression, hysteresis loops during unloading–reloading, just mention the most popular ones [4–12].

In rock mechanics, investigations on constitutive modelings were first devoted to fitting rock strength in terms of either principal stresses or their invariants but without deformation analyses. In that context, several well-known rock failure criteria have been proposed e.g. [13–16] and later incorporated in the framework of the classical plasticity theory for describing inelastic deformation. By nature, elastoplastic models are capable of modeling strain hardening and even strain softening phenomena by using additional loading history-dependent functions [17,18]. However, material degradation and inelastic deformation are more popularly characterized separately [19,20]. Precisely, the former is framed by the continuum damage theory while inelastic deformation by the plasticity theory. In general, these two mechanisms usually involved in rocks and characterized by means of two internal variables are competing and strongly coupled to each other. In that coupling process, it is generally very difficult to predict analytically rock strength from a set of constitutive equations, especially in micromechanical and poromechanical setting [8,12]. On this aspect, recent advances on micro–macro modeling of cracked solids have paved a promising way to study more rationally the failure of brittle or quasi-brittle rocks [8,11,12,21].

The purpose of this paper is twofold. The first one is to derive failure functions based on micromechanics-based unilateral elastic

* Address: College of Civil and Transportation Engineering, Hohai University, Nanjing, China.

E-mail address: qizhi.zhu@gmail.com

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damage as well as damage-friction coupling formulations for both dry and saturated cracked media. As the starting-point of our researches along the line, we are particularly interested to deliver a new way to derive a new failure criterion from microscopic cracking mechanisms. In order to simplify constitutive formulations, isotropic assumptions have been made on crack growth and inelastic deformation [20]. The coupling model is presented in a more elegant and compact manner. Strength prediction of brittle rocks is achieved for both the cases of open and closed cracks under either dry or saturated conditions. We also address some relevant theoretical issues, such as formulation of a proper damage criterion and calibration of the model's parameters in combination of conventional experimental data. It is largely admitted that rational determination of damage evolution law is still one of the open issues in Continuum Damage Mechanics. Although strain energy release rate based damage criteria have been widely adopted as a compromise, the damage resistance function involved therein is still empirical by now. In the second part, also in the coupling context, original contributions will be devoted to elucidating some basic features that a damage resistance should possess. For practical application, the method of calibrating the model's parameters is also presented through critical analyses on stress–strain curves of Lac du Bonnet granite. Finally, for a first stage of validation, the proposed failure criterion is applied to Westerly granite under triaxial loadings.

It is emphasized that although limited to isotropic formulations, the principles and derivation procedure presented in this work can be adapted easily to the case of anisotropic orientation-dependent crack models [8,12].

2. Formulation of a micromechanical damage-friction model

The homogenization-based isotropic damage-friction model developed by Zhu et al. [20] for cracked brittle solids is reformulated in a more element form. Here, the free energy in the matrix-cracks system is derived by applying the principle of stress and energy equivalences in lieu of the strain decomposition technique [9,11].

2.1. Free energy

Quasi-brittle rocks weakened by microcracks are modeled as heterogeneous materials and investigated in a combined homogenization/thermodynamic framework. The relevant representative elementary volume (REV) consists of a solid matrix phase and a large number of microcracks randomly oriented and distributed in the former. The matrix is commonly assumed to be linearly elastic with stiffness tensor $\mathbb{C}^m$. All microcracks are simplified to be penny-shaped and approximated as ellipsoidal inclusions. Each crack is in fact a discontinuity across which the displacement field suffers a jump. It is then possible to decompose the macroscopic strain $\boldsymbol{\varepsilon}$ into two parts, $\boldsymbol{\varepsilon}^m$ and $\boldsymbol{\varepsilon}^c$, related to the respective contributions by the matrix phase and microcracks

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^m + \boldsymbol{\varepsilon}^c \tag{1}$$

Accordingly, the macroscopic stress $\boldsymbol{\sigma}$ takes the classical form

$$\boldsymbol{\sigma} = \mathbb{C}^m : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^c) \tag{2}$$

For open cracks, damage by crack growth is regarded as the unique energy dissipation mechanism. Within the linear homogenization framework, the inelastic stain $\boldsymbol{\varepsilon}^c$ is related linearly to the macroscopic strain applied onto the boundary, such that

$$\boldsymbol{\varepsilon}^c = \mathbb{A}^c : \boldsymbol{\varepsilon} \tag{3}$$

where $\mathbb{A}^c$ is interpreted as the global strain concentration tensor for crack phase. The determination of $\mathbb{A}^c$ is dependent directly on the choice of homogenization scheme. When limited to elastic microcracking, the effective properties predicted by the Mori–Tanaka homogenization scheme [22,23] has close links with the results established in the Linear Elastic Fracture Mechanics [4,8]. For this reason, the Mori–Tanaka scheme is adopted for later formulations. When the Mori–Tanaka method is concerned, the following relation is obtained under isotropic assumptions [20]

$$\boldsymbol{\varepsilon}^c = \left(\frac{\eta_1\omega}{1+\eta_1\omega}\mathbb{J} + \frac{\eta_2\omega}{1+\eta_2\omega}\mathbb{K}\right) : \boldsymbol{\varepsilon} \tag{4}$$

where ω is the crack density parameter and servers here as the damage variable, η_1 and η_2 are two constants only function of the Poisson's ratio ν^m of the matrix phase, such that $\eta_1 = \frac{16}{9}\frac{1-(\nu^m)^2}{1-2\nu^m}$ and $\eta_2 = \frac{32}{45}\frac{(1-\nu^m)(5-\nu^m)}{2-\nu^m}$. $\mathbb{J}$ and $\mathbb{K}$ are the fourth order isotropic and deviatoric projectors, respectively. By using the second order identity tensor $\boldsymbol{\delta}$, the components of $\mathbb{J}$ and $\mathbb{K}$ are $J_{ijkl} = \frac{1}{3}\delta_{ij}\delta_{kl}$ and $K_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - J_{ijkl}$, respectively. Substitution of Eq. (4) into Eq. (2) leads to the effective stiffness tensor,

$$\mathbb{C}^{hom} = \frac{1}{1+\eta_1\omega}3k^m\mathbb{J} + \frac{1}{1+\eta_2\omega}2\mu^m\mathbb{K} \tag{5}$$

Equivalently to Eq. (2), the macroscopic stress $\boldsymbol{\sigma}$ can be reformulated by means of $\mathbb{C}^{hom}$

$$\boldsymbol{\sigma} = \mathbb{C}^{hom} : \boldsymbol{\varepsilon} \tag{6}$$

On the other hand, it has been proved that the free energy of the matrix-cracks system takes the general form [11,20]

$$W = \frac{1}{2}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^c) : \mathbb{C}^m : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^c) + \frac{1}{2}\boldsymbol{\varepsilon}^c : \mathbb{C}^b : \boldsymbol{\varepsilon}^c \tag{7}$$

where the second term on the right hand side represents the stored energy due to the existence of microcracks. It is noted that Eq. (7) is valid for both the cases of open and closed cracks. For open cracks, the equivalence between the form $W = \frac{1}{2}\boldsymbol{\varepsilon} : \mathbb{C}^{hom} : \boldsymbol{\varepsilon}$ and Eq. (7) leads to the explicit expression of $\mathbb{C}^b$

$$\mathbb{C}^b = \frac{1}{\eta_1\omega}3k^m\mathbb{J} + \frac{1}{\eta_2\omega}2\mu^m\mathbb{K} \tag{8}$$

When the matrix is weakened by closed cracks, there are generally two coupling dissipative mechanisms: damage evolution due to progressive propagation of microcracks and frictional sliding along cracks accompanied by volumetric dilatancy. One of the significant consequences by frictional sliding is that the inelastic strain $\boldsymbol{\varepsilon}^c$, which takes the explicit form (3) in the case of open cracks, needs to be evaluated incrementally. In this setting, both $\boldsymbol{\varepsilon}^c$ and ω should be treated as internal variables and the former can be handled within the classical theory of plasticity.

2.2. State equations

Given the free energy W, the state equations, also termed as the thermodynamic forces, are derived by applying standard differentiation of W with respect to the internal variables. For the damage variable ω, we obtain from Eq. (7) the damage conjugate force

$$F_\omega = -\frac{\partial W}{\partial \omega} = \frac{1}{2}\boldsymbol{\varepsilon}^c : \left(\frac{1}{\eta_1\omega^2}3k^m\mathbb{J} + \frac{1}{\eta_2\omega^2}2\mu^m\mathbb{K}\right) : \boldsymbol{\varepsilon}^c \tag{9}$$

The thermodynamic force associated with $\boldsymbol{\varepsilon}^c$ is obtained for closed frictional cracks

$$\boldsymbol{\sigma}^c = -\frac{\partial W}{\partial \boldsymbol{\varepsilon}^c} = \boldsymbol{\sigma} - \mathbb{C}^b : \boldsymbol{\varepsilon}^c \tag{10}$$

It is emphasize that $\boldsymbol{\sigma} = \partial W/\partial\boldsymbol{\varepsilon} = \mathbb{C}^m : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^c)$ represents the macroscopic stress, while $\boldsymbol{\sigma}^c$ is the local stress acting onto cracks.