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Research Paper

# Evaluating slope stability uncertainty using coupled Markov chain

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A B S T R A C T

Geological uncertainty appears in the form of one soil layer embedded in another or the inclusion of pockets of different soil type within a more uniform soil mass. Uncertainty in factor of safety (FS) and probability of failure (Pf) of slope induced by the geological uncertainty is not well studied in the past. This paper presents one approach to evaluate the uncertainty in FS and Pf of slope in the presence of geological uncertainty using borehole data. The geological uncertainty is simulated by an efficient coupled Markov chain (CMC) model. Slope stability analysis is then conducted based on the simulated heterogeneous soils. Effect of borehole layout schemes on uncertainty evaluation of FS and Pf is investigated. The results show that borehole within influence zone of the slope is essential for a precise evaluation of FS statistics and Pf. The mean of FS will converge to the correct answer as the borehole number increases.


## 1. Introduction

It is well known that soil heterogeneity or spatial variability of soil properties plays a significant role in the performance of geotechnical systems such as a slope (e.g. [6,7,10,12,17,19,25,26,28]). At present, studies on spatial variability in geotechnical engineering mainly focused on inherent variability within one nominally homogeneous layer [11,14,16,20,22]. The inherent variability is the variation of soil property parameters from one point to another due to different deposition conditions and loading histories [4,21]. It occurs in a soil mass that belongs to the same material type. However, another form of soil heterogeneity, namely geological uncertainty, also exists in reality (e.g. [5,9]). It appears in the form of one soil layer embedded in another or inclusion of pockets of different soil type within a more uniform soil mass [4]. Some attention has been paid to this kind of uncertainty. For example, Tang et al. [27] introduced a renewal process to describe the probabilistic nature of a soil stratum consisting of two distinct material types. Halim [8] evaluated the reliability of geotechnical systems considering the uncertainty of geological anomaly. Herein the geological anomaly refers to the case of pockets of different soil type included within a more uniform soil mass. The occurrence of geological anomalies in space is modeled by a Poisson process in this study. Kohno et al. [15] studied the system reliability of a tunnel running through two rock types. Similarly, the occurrence of the less dominant rock in the two was also modeled by a Poisson process. The limitations of these study are quite obvious. Both the renewal process and the Poisson process can only model the two simplest forms of geological uncertainty. Only two types of soil are involved in these forms. The more general form, namely layers with more than two material types embedding each other, cannot be handled by these two processes.

In reality, geological uncertainty typically involves more than two soil types embedding each other in a layered profile (e.g. [5,9]). There are very limited studies on how this form of uncertainty affects the factor of safety (FS) and probability of failure (Pf) of a slope. This paper aims to evaluate the uncertainty in FS and Pf of a slope in the presence of geological uncertainty using borehole data. Coupled Markov chain (CMC) is an effective model to simulated geological uncertainty. To facilitate the application of this model in geotechnical practice, Qi et al. [23] proposed a practical method to estimate one key input of the CMC model, i.e. horizontal transition probability matrix (HTPM). Based on this method, this paper applies the CMC model to a slope problem. The borehole database for the Perth Central Business District, Western Australia, is adopted to simulate the geological uncertainty conditional on known stratigraphy given in the boreholes. Three types of soils (clay, silt and sand) are present in this database. Inherent variability is not considered within each layer. Based on the simulated heterogeneous soils, the FS of slope can be calculated using the finite element-strength reduction method. Monte Carlo simulation of slope stability analysis is conducted using different

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borehole layout schemes. The effect of borehole layout schemes on the FS statistics (including mean and standard deviation) and Pf is analyzed. To overcome the limitation of too less borehole number, some virtual boreholes are created to further investigate the role of borehole number and borehole location in evaluating the uncertainty in FS of slope.

## 2. Coupled Markov chain model

Coupled Markov chain (CMC) is a random process model, which can simulate the geological uncertainty involving more than two types of soils embedding each other [3]. The model is theoretically simple, explicit and computationally efficient. It is a coupled product of two one-dimensional Markov chains. One describes the sequence of soil states in horizontal direction, and the other in vertical direction. Herein the soil state refers to soil type, such as sand, clay and silt. For each one-dimensional Markov chain, the probability of transitions between different soil states are denoted by one transition probability matrix, i.e. horizontal transition probability matrix (HTPM, $\boldsymbol{P^h}$) for the horizontal Markov chain, and vertical transition probability matrix (VTPM, $\boldsymbol{P^v}$) for the vertical Markov chain. Both matrices have a size of $m * m$, where $m$ ($m \geqslant 2$) is the total number of soil state involved. For example, the element in $i$th row, $j$th column of VTPM, namely $p^v_{ij}$, denotes the probability of transition from soil state $i$ ($S_i$) to soil state $j$ ($S_j$) in vertical direction.

The basic idea of CMC is as follows. As shown in Fig. 1, the domain to be modeled is discretized into a number of cells with the same size. The state of cell $(i,j)$ ($i > 1$, $i$ = column number; $j > 1$, $j$ = row number) depends on the states of the cells on the top [cell $(i, j-1)$], left [cell $(i-1, j)$] and rightmost [cell $(N_x,j)$, $N_x$ = the sum of cell columns] of the current cell. Soil states on the leftmost column [i.e. the cells $(1,j)$, $j = 1,\ldots, N_z$, $N_z$ = the sum of cell rows] (considered as left boundary of the simulation domain), rightmost column [i.e. the cells $(N_x,j)$, $j = 1,\ldots, N_z$] (considered as right boundary of the simulation domain) and top row [i.e. the cells $(i,1)$, $i = 1,\ldots, N_x$] are fixed. The former two are revealed by two boreholes while the latter is directly observable from the ground surface. They can be used as conditional information to simulate the states of the other cells inside the domain. The dependence of the cell states is described in terms of transition probabilities as

$$p_{lr,k|q} = \frac{p^h_{lk} p^{h(N_x-i)}_{kq} p^v_{rk}}{\sum_{f=1}^{m} p^h_{lf} p^{h(N_x-i)}_{fq} p^v_{rf}} \quad (1)$$

where $p_{lr,k|q}$ is the probability that cell $(i,j)$ is in state $S_k$, given that cell $(i-1,j)$, $(i,j-1)$ and $(N_x,j)$ is in state $S_l$, $S_r$ and $S_q$; $p^h_{lk}$ and $p^v_{rk}$ are the corresponding elements of the horizontal and vertical transition probability matrices, $\boldsymbol{P^h}$ and $\boldsymbol{P^v}$; $p^{h(N_x-i)}_{kq}$ [$p^{h(N_x-i)}_{fq}$] is the probability of transition from $S_k$ ($S_f$) to $S_q$ in $(N_x - i)$ steps in the horizontal direction. It is the corresponding elements of $(\boldsymbol{P^h})^{(Nx-i)}$, i.e. the matrix obtained by multiplying HTPM by itself $(N_x - i)$ times.

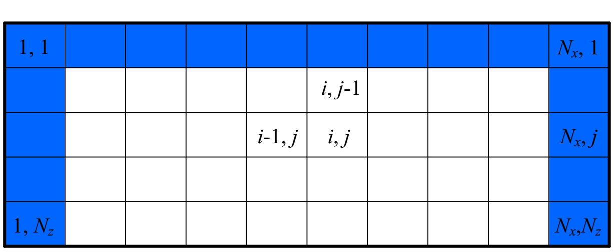


**Fig. 1.** Numbering system in a two-dimensional domain for the coupled Markov chain.

## 3. Borehole data

Some borehole data from Central Business District, Perth, Western Australia are collected for geological uncertainty simulation in this paper. The relative location and stratigraphy of the boreholes are plotted in Fig. 2. As shown by Fig. 2(a), the boreholes are scattered distributed within a 70 m × 100 m area. To construct a two-dimensional model of slope, all the boreholes need to be projected to a line that is parallel to the sliding direction of the slope. Since there are no real slopes nearby the borehole area, a projection line parallel to $x$ axis is assumed to be the sliding direction of a slope. The locations of the projected borehole are obtained as shown in Fig. 2(a). For brevity, the boreholes from left to right (i.e. boreholes D76′, D77′, D75′, D78′, D23′, D20′) are re-labeled as boreholes 1, 2, 3, 4, 5, 6 from hereon. Fig. 2(b) illustrates the stratigraphy revealed by the boreholes. As shown by Fig. 2(b), three types of soil (i.e. clay, silt and sand) are involved. The material in the top layer is sand in all boreholes. Borehole lengths vary from 22.2 m to 29.2 m. The maximum interval distance between the boreholes in $x$ direction is 70 m. The minimum thickness of the geological unit revealed by boreholes is 0.3 m [see borehole 4 in Fig. 2(b)].

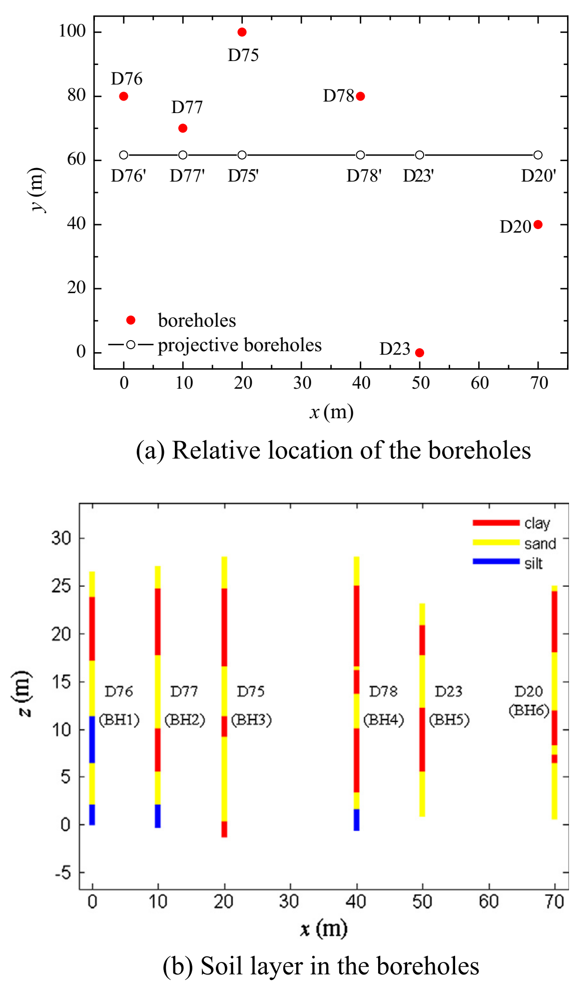


**Fig. 2.** The relative location and stratigraphy of the boreholes in Perth city, Australia.