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Research Paper

An analytical solution for excited pile vibrations with variable section impedance in the time domain and its engineering application

Liu Gao, Kuihua Wang *, Si Xiao, Zhenya Li, Juntao Wu

Key Laboratory of Soft Soils and Geoenvironmental Engineering, Ministry of Education (Zhejiang University), Hangzhou 310058, China
Research Center of Coastal and Urban Geotechnical Engineering, Zhejiang University, Hangzhou 310058, China

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ABSTRACT

Through pile defect generalization, a definite solution to the problem of longitudinal vibrations in a finite pile lying on an elastic stratum with variable section impedance is established using the δ function. By separating variables and applying the boundary and continuity conditions of the pile–soil system, the characteristic equation for the problem is derived, and the corresponding eigenvalues are determined through numerical calculations. Then, the analytical solution for a pile subjected to steady sinusoidal excitation in the time domain is derived. The amplitude–frequency curves are analysed, and conclusions are drawn. An analytical solution for a pile subjected to transient excitation in the time domain is obtained using the principle of superposition. Then, a parametric study is conducted to investigate the characteristics of signals reflected from a defect, yielding several conclusions that are important for guiding practical defect detection. Finally, two engineering projects are described, and the theoretical and measured curves are compared.

1. Introduction

Pile foundations are commonly used in construction engineering to ensure sufficient strength and settlement tolerance. The vibration characteristics of a pile subjected to a dynamic load have been investigated thoroughly, and the resulting vibration theories provide a basis for various methods of defect detection and seismic design for piles. Using the general Winkler model, Novak studied the dynamic response of a pile subjected to vertical and horizontal excitations in the frequency domain and proposed a method of calculating the spring and damping coefficients [1]. Using three Voigt models connected in series to simulate the inhomogeneity of the surrounding soil, Nogami and Konagai converted the solution for a pile's vibration in the frequency domain into the time domain and comprehensively investigated the bending response of a pile to a transverse load [2–4]. Baranov proposed the first method for calculating the excited vibration of an embedded foundation based on the plane strain model [5]. Subsequently, more thorough studies based on the plane strain model have been conducted [6–9]. Over the past two decades, research on the coupled vibration of a pile and soil based on a three-dimensional continuum model has advanced significantly [10–14]. With developments in computer technology, the finite and boundary element methods (FEM and BEM, respectively), which are used for numerical modelling, have also progressed significantly; they are now able to account for the inhomogeneity and nonlinearity of the surrounding soil, the discontinuity at the pile–soil interface and the dynamic response of groups of piles [15–19].

Considering the effects of the surrounding soil, Koten and Middendrop proposed an analytical solution for an infinite pile that takes the form of an integral expression containing a Bessel function with an imaginary argument [20]. In a real project, the focus of the defect detection task is the detection of a defective pile. Therefore, knowledge of the dynamic characteristics of a defective pile has considerable theoretical and engineering value. Although a significant amount of research has been conducted on the subject, yielding the fruitful discoveries described above, analytical methods have not advanced because of their mathematical complexity. Most analytical solutions in the time domain have been obtained using a numerical Laplace inversion procedure. An analytical solution in the time domain reflects the relationship between the independent variable and dependent variable more concisely and clearly than do semi-analytical and numerical solutions. Therefore, finding an analytical solution in the time domain is of great theoretical importance. This paper focuses on this issue, and an

* Corresponding author at: Research Center of Coastal and Urban Geotechnical Engineering, Zhejiang University, Hangzhou 310058, China. Tel./fax: +86 571 88208708.

E-mail address: zdwkh0618@zju.edu.cn (K. Wang).

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analytical solution for excited vibrations in a pile with variable section impedance in the time domain is proposed.

By applying defect generalization, a defective pile is modelled as a pile with the same cross-sectional area. By means of the δ function, a one-dimensional wave equation is used to analyse the problem. By combining this equation with the boundary and continuity conditions, a definite problem is established to describe the dynamic characteristics of a defective pile subjected to steady sinusoidal excitations from the top. Using the separation of variables, the characteristic equation is derived, and the corresponding eigenvalues are calculated numerically. Using the orthogonality of the eigenfunctions, an analytical solution for the vibration of a pile subjected to steady sinusoidal excitations in the time domain applied at its top is expressed in series form. The parameters of the amplitude–frequency curve are studied. Then, an analytical solution for the vibration of a pile subjected to transient excitation in the time domain applied at its top is obtained using the principle of superposition, and the parameters are studied. A method is proposed to confirm the actual burial depth of a variable-impedance interface based on the results of the fitting process, and the results obtained using this method are found to be in good agreement with the true parameters, thereby confirming the rationality of the analytical solution.

2. Pile defect generalization and the establishment of a definite problem

2.1. Pile defect generalization

Pile defects may belong to any of the following categories: neck collapse, neck expansion, concrete segregation, cracking, and mud clamping. Pile defects are extremely complex in practice; however, in this paper, they are generalized to allow a one-dimensional wave equation to be used to analyse the problem. Defects of all types are treated as having the same cross-sectional area and density as a normal pile segment but different Young's moduli. The cross-sectional acoustic impedance of such a generalized segment is equal to that of the original defect in the pile segment. The pile length is also adjusted such that the same amount of time is required for an incident impulse to propagate from the pile top to the pile tip in the generalized model as is required in the original pile. The generalized pile–soil geometrical model is depicted in Fig. 1.

2.2. Establishment of the definite problem

The following assumptions are adopted for this analysis:

(1) The length of the pile is L, and its mass density is ρ. The pile perimeter is C. The pile shaft is divided into three segments according to the acoustic impedance. The lengths of these segments are h_1, $h_2 - h_1$, and $L - h_2$, the Young's moduli are E_1, E_2, and E_3, and the elastic wave velocities are C_1, C_2, and C_3. The cross-sectional area is S.
(2) Only vertical vibrations are considered, and the pile–soil system is subjected to linear elastic deformations. The surrounding soil is represented by a spring with a spring constant k_s and a damper with coefficient η_s connected in parallel; the bearing stratum is simulated by linearly distributed springs with a spring constant k_{toe}.
(3) The exciting force at the top of the pile is evenly distributed, and its amplitude varies with time.

The fundamental equation for each pile segment is derived by transforming the excitation at the top of the pile into a load

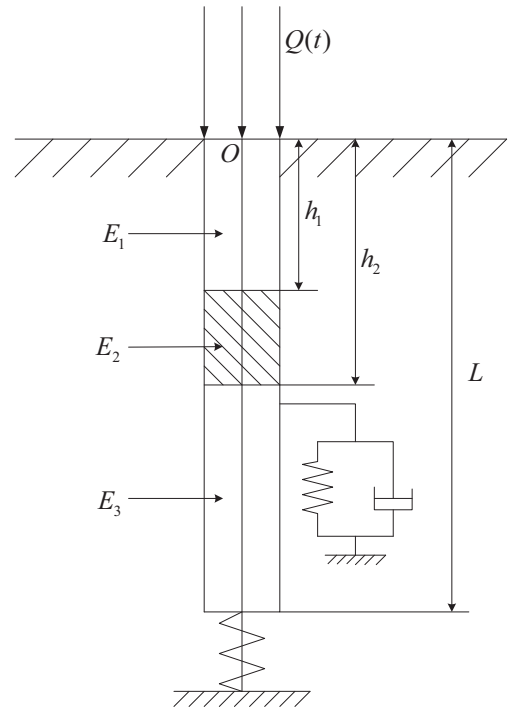

Fig. 1. Generalized pile–soil geometrical model.

distributed along the pile shaft using a δ function. The continuity conditions at the variable-impedance interface are as follows:

(1) The displacements of particles across the interface are equal.
(2) The stresses on both sides of the interface are equal.

Combining these conditions with the initial and boundary conditions results in the following definite problem:

$$C_1^2\frac{\partial^2 u_1}{\partial x^2} = \frac{\partial^2 u_1}{\partial t^2} + A\frac{\partial u_1}{\partial t} + Bu_1 - \frac{Q(t)}{\rho S}\delta(x); \quad 0 \leqslant x \leqslant h_1; \tag{1}$$

$$u_1 = u_2; \quad E_1 S\frac{\partial u_1}{\partial x} = E_2 S\frac{\partial u_2}{\partial x}; \quad x = h_1; \tag{2}$$

$$C_2^2\frac{\partial^2 u_2}{\partial x^2} = \frac{\partial^2 u_2}{\partial t^2} + A\frac{\partial u_2}{\partial t} + Bu_2; \quad h_1 \leqslant x \leqslant h_2; \tag{3}$$

$$u_2 = u_3; \quad E_2 S\frac{\partial u_2}{\partial x} = E_3 S\frac{\partial u_3}{\partial x}; \quad x = h_2; \tag{4}$$

$$C_3^2\frac{\partial^2 u_3}{\partial x^2} = \frac{\partial^2 u_3}{\partial t^2} + A\frac{\partial u_3}{\partial t} + Bu_3; \quad h_2 \leqslant x \leqslant L; \tag{5}$$

$$u_1(x,0) = 0; \quad \left.\frac{\partial u_1}{\partial t}\right|_{t=0} = 0; \quad 0 \leqslant x \leqslant h_1 \tag{6}$$

$$u_2(x,0) = 0; \quad \left.\frac{\partial u_2}{\partial t}\right|_{t=0} = 0; \quad h_1 \leqslant x \leqslant h_2; \tag{7}$$

$$u_3(x,0) = 0; \quad \left.\frac{\partial u_3}{\partial t}\right|_{t=0} = 0; \quad h_2 \leqslant x \leqslant L; \tag{8}$$

$$\left.\frac{\partial u_1(x,t)}{\partial x}\right|_{x=0} = 0; \quad \left.\left(\frac{\partial u_3}{\partial x} + R'u_3\right)\right|_{x=L} = 0; \tag{9}$$

here, u_1, u_2 and u_3 denote the displacements of pile segments I, II and III, respectively. In addition, $A = \frac{\eta_s}{\rho}\cdot\frac{C}{S}$, $B = \frac{k_s}{\rho}\cdot\frac{C}{S}$, $R' = \frac{k_{toe}}{E_3}$, and $C_i = \sqrt{\frac{E_i}{\rho}}(i = 1, 2, 3)$.

3. Solution to the definite problem

This section proposes a transient analytical solution to the definite problem presented above, which lays the foundation for the subsequent analysis.