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Research Paper

Simulating the Poisson effect in lattice models of elastic continua

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ABSTRACT

Lattice models provide discontinuous approximations of the displacement field over the computational domain, which facilitates the modeling of fracture and other discontinuous phenomena. By discretizing the domain with two-node elements, however, ordinary lattice models cannot simulate the Poisson effect in a local (intra-element) sense, which is problematic for some types of analyses. Furthermore, such methods are limited in the range of Poisson ratio values that can be simulated. We present a new approach to remedy such known, yet underappreciated, shortcomings of lattice models. In this approach, the Poisson effect is modeled through the introduction of fictitious stresses into a regular lattice. Capabilities of the new approach are demonstrated through compressive test simulations of homogeneous and heterogeneous materials. The simulation results are compared with theory and those of continuum finite element models. The comparisons show good agreement for arbitrary Poisson ratios (including $\nu \geqslant 1/3$) with respect to nodal displacement, intra-element stress, and nodal stress. This form of discrete method, supplemented by the proposed fictitious measures of stress, retains the simplicity of collections of two-node elements.

1. Introduction

Lattice models are attractive for simulating the fracturing of various materials, particularly when fracture development is affected by material structure or other forms of heterogeneity present at the scale of discretization. Lattice models are typically based on a set of nodes and their interconnection via primitive, one-dimensional (1-D) elements. Such models, which include some types of particle models, can be viewed as mechanical analogues of the equations of continuum mechanics [1,16,21,31]. The nodes can be arranged in regular or irregular patterns. Continuum properties are obtained, in an approximate sense, through appropriate assignments of the element properties. As described herein, however, lattice models are limited in their abilities to represent local stress conditions, particularly with respect to the influence of Poisson's ratio [6,18,12]. Proper representation of the Poisson effect is an essential ingredient within most rock mechanics simulations, including those affected by multiaxial stress conditions or material heterogeneity.

Beginning with the work of Hrennikoff [19], a variety of discrete methods have been developed to represent continua as collections of particles or lattice structures. Particle-based methods, including the discrete element method [9], are used to simulate the interaction of discrete features and their collective influence on the behavior of geological systems. Micro-mechanical parameters used in the discrete elements (i.e., springs or bonds between the particles) can be determined, through calibration with laboratory results, to represent macroscopic material behavior [38]. Random particle models are also used to simulate fracture behavior of other geomaterials such as concrete [4,10]. Macroscopic representation of the Poisson effect is accomplished by adjusting the ratio of the average strain between the longitudinal and transverse directions [10,11]. Lattice models are another means for studying elasticity and breakdown of a variety of materials and structures [17,33,22,15]. Global representation of Young's modulus and Poisson's ratio can be obtained by adjusting longitudinal and transverse dimensions, or stiffnesses, of the lattice beam elements [8,3,36]. Whereas such models simulate the Poisson ratio in a global sense, inaccuracies are present at the elemental level, which can be viewed as an artificial form of heterogeneity that is not present in most models constructed from continuum elements. Moreover, direct linkages between input mechanical parameters and experimental measurements are difficult to establish for discrete methods [31].

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The elasticity of discrete methods, without using free parameters, has been studied. Griffiths and Mustoe [16] relate the elastic constants, Young's modulus and Poisson's ratio, and the spring stiffnesses through an approach based on strain energy density. Such relations are used herein for comparison purposes. Liu et al. [25] derive similar relationships to model failure behaviors such as breaking displacement, shear resistance, and coefficient of friction. Alternatively, three-node discrete element models have been developed to accommodate a volumetric constitutive relation. Hori et al. [18] proposed a discrete-type finite element model based on the use of discontinuous shape functions for each node. Cusatis and Schauffert [12] developed an integrated framework between discrete and continuum methods to overcome the disadvantages of discrete methods. A local representation of both Young's modulus and Poisson's ratio was obtained by a hybrid system, in which a planar lattice is combined with constant strain triangle finite elements [6]. Although these discrete approaches accurately represent the Poisson effect with a set of discrete springs, or in conjunction with finite elements, the simplicity of two-node elements (as a means for modeling material breakdown) is compromised. Munjiza [27] developed a combined finite–discrete element method (FDEM), in which elasticity calculations are based on continuum finite element methods, and discontinuous behavior is represented by a discrete method. Whereas the transition from continuous to discontinuous behavior needs proper attention [5,29,30,34], FDEM capably simulates both elasticity and failure processes of geomaterials, as demonstrated through comparisons with theory and laboratory studies [26,24]. Munjiza et al. [28] cover several methods that describe physical systems using discrete entities.

This paper calls attention to significant shortcomings of discontinuous (lattice) models with respect to simulating stress conditions within elastic continua. In particular, models constructed with discrete, two-node, elements do not provide a local representation of the Poisson effect. Furthermore, such models do not accommodate the full range of Poisson's ratio, nor even the range exhibited by some rocks [14]. A new approach is presented to address these known, yet underappreciated, shortcomings. Transverse strains, based on fictitious measures of principal stress calculated at the nodal points, are iteratively introduced to accurately represent the Poisson effect within a regular lattice. With the proposed approach, element stiffnesses are based directly on the material properties (i.e. Young's modulus and Poisson's ratio), such that calibration processes are not necessary. To demonstrate the accuracy of the proposed approach, simulation results for homogeneous systems under uniform loading are compared with both analytical solutions and practical relationships, which have been widely used to determine the spring constants of discrete methods. Comparisons are made for intra-element stress, nodal stress, and nodal displacement. Thereafter, the accurate modeling of multi-phase systems is demonstrated through comparisons with finite element results.

2. Modeling of elastic continua: limitations of lattice models

Lattice models are based on discontinuous approximations of the field variable over the computational domain. This facilitates the modeling of fracture development and other discontinuous phenomena. However, there are significant shortcomings of lattice models with respect to representing local stress conditions. For example, for the boundary conditions and loading shown in Fig. 1a, conventional continuum approaches predict uniaxial compressive stress at any point within the domain. All normal stress components are either compressive or have zero magnitude. Lateral straining (i.e., the Poisson effect) occurs, in accordance with theory. Consider a regular truss network, configured as shown in Fig. 1b and likewise loaded in compression. It exhibits lateral straining, but only at a fixed proportion of the vertical compressive strain. Furthermore, the lateral truss elements are in tension, which disagrees with conventional theories of elasticity. Such truss networks exhibit vertical cracking when the lateral tension reaches the prescribed tensile strength of the material. Whereas cracking parallel to the direction of compressive loading has been observed during physical testing, such cracking is typically a consequence of finer-scale material heterogeneity and is more appropriately related to strain capacity. Moreover, the truss element forces depend on the orientation of the truss network with respect to the direction of loading.

By supplementing the truss elements with shear and rotational stiffnesses, a range of macroscopic Poisson ratio can be simulated. One such lattice model is presented in the following section. The macroscopic Poisson ratio can be controlled by adjusting the relative magnitudes of the axial and shear stiffnesses. However, such lattice models provide a flawed representation of the Poisson effect: under uniaxial compressive loading, tension is wrongly produced in the orthogonal direction.

3. Lattice model formulation

Hereafter, a specific form of lattice model, based on the rigid-body-spring concept of Kawai [23], is used to discretely represent elastic continua. This approach has been used to simulate elasticity and breakdown of a variety of materials [7,13,2]. For a triangular array of nodal points, the lattice geometry is shown in Fig. 2a. In this study, nodal connectivity is prescribed and remains constant throughout the analysis: contact modeling used in the Distinct Element Method (DEM) is not considered. For this 2-D case, each node has two translational and one rotational degrees of freedom. Each element ij is composed of a zero-size spring set that is connected to nodes i and j via rigid links. The spring set is formed from two axial (normal and tangential) springs, k_n and k_t, and one rotational spring, k_φ, as shown in Fig. 2b. The spring coefficients are assigned according to

$$k_t = \alpha_1 k_n = \alpha_1 \alpha_2 E \frac{A_{ij}}{h_{ij}}, \quad k_\varphi = E \frac{I_{ij}}{h_{ij}} \tag{1}$$

in which E is the Young's modulus, A_{ij} is the area of the facet common to nodes i and j (Fig. 2a), h_{ij} is the distance between the same nodes, and I_{ij} is the second moment of area A_{ij}. By adjusting α_1 and α_2, macroscopic modeling of both elastic constants (E and Poisson ratio, ν) is possible.

By equating strain energy densities of an elastic continuum (in plane stress) and a regular triangular lattice, the spring coefficients are related to the elastic constants as follows [16]:

$$k_n = \frac{E}{\sqrt{3}(1-\nu)}, \quad k_t = \frac{E(1-3\nu)}{\sqrt{3}(1-\nu^2)} \tag{2}$$

where the range of Poisson ratio is limited to $-1 < \nu < 1/3$. Similar formulations can be found elsewhere [20,25] and are widely used to determine the spring constants of discrete methods. For a particular value of the Poisson ratio, it is possible to rewrite Eq. (2) in terms of α_1 and α_2. For example, for $\nu = 0.2$, α_1 and α_2 become 1.25 and 1.25/3, respectively, which are used herein for comparison purposes. Whereas strain conditions obtained by Eq. (2) for regular lattices have been validated, the local stress conditions have not been discussed [16]. The following section introduces existing procedures to determine local stress conditions from the lattice structure, and a new approach to accurately represent the Poisson effect.