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Research Paper

Nonlinear analyses of laterally loaded piles – A semi-analytical approach

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ABSTRACT

This paper describes a semi-analytical, continuum mechanics-based method of calculating the non-linear response of pile foundations subjected to lateral loads. The displacement field in the soil is obtained as the product of a pile deflection function and displacement decay functions. Based on the incremental form of the virtual work, a system of coupled differential equations for the pile deflection and decay functions is derived and then solved by using the finite difference method, producing pile deflections, shear forces and bending moments, as well as displacements within the soil domain. The soil is modeled as an elasto-plastic material. The analysis is computationally efficient and produces results that are in good agreement with those from 3D finite element modeling with the same constitutive model for the soil. The method models laterally loaded pile response more realistically than the traditional *p–y* method at a comparable computational cost. The method is substantially more efficient than 3D finite element simulations. The proposed method offers a good alternative to the *p–y* method in contexts in which fast calculation of lateral pile response is required, such as in the design of piles for offshore wind farms.

1. Introduction

Design of laterally loaded piles often relies on the *p–y* method in routine engineering practice due to its versatility and speed. The pile is assumed to be an Euler–Bernoulli beam acted upon by a point load (and/or a bending moment) at one end (the pile head) and soil reactions along the pile length. The soil reactions are assumed applied at discrete locations and are modeled as a series of independent, non-linear, one-dimensional springs. Considering one spring per unit length of pile, the force p per unit length of pile transferred to each spring is assumed to be a non-linear function of the lateral deflection y of the pile at the location of the spring.

The *p–y* method has certain limitations that result from its rather simple modeling of the pile–soil interaction problem. For example, the diameter (or width) of the pile is the only input parameter used in current *p–y* curves that takes into account the effect of the pile. However, it has been found that the pile geometry (both shape and dimensions) and bending stiffness EI affect the shape and magnitude of *p–y* curves [1]. Some of the parameters (for instance, the dimensionless empirical constant J in the API soft clay criterion [2]) in the *p–y* method are arbitrarily determined by the users based on their experience rather than by fundamental considerations. Thus, significant judgment enters the definition of *p–y* curves, and site calibration is almost always needed if realistic predictions are desired.

Three-dimensional numerical analyses, such as the finite element method or the finite difference method [3–12] would produce the most realistic solutions of the laterally loaded pile problem when appropriate soil models are used and the correct soil state is considered in the soil around the pile. Yet, these methods are not often used in routine geotechnical engineering practice due to the modeling knowledge required to properly use these techniques and the relatively long problem set up and computation time. This means that semi-analytical solutions that can be shown to produce accurate, realistic results in computation times comparable to those of the *p–y* method have a role in the practice of foundation engineering. This paper presents one such method that can be used for piles in elasto-plastic soil.

By linking the soil displacement directly to the pile deflection and describing the soil–pile system using energy principles, analytical or semi-analytical energy-based methods have been developed and improved through the years; the applicability of these methods has been progressively extended from piles in uniform soil [13,14] to piles in layered soil profiles [15–18] and from single piles [19] to pile groups [20]. These analytical or semi-analytical solutions have proven to be reliable and computationally efficient, but all treated the soil as an elastic material, making it difficult to use them in practice without considerable judgment.

A new formulation is proposed in this paper to accommodate the implementation of an elasto-plastic constitutive model for the soil while maintaining the efficiency of the previous

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formulations. The new formulation enables us to make more realistic predictions of the non-linear, three-dimensional response of the soil–pile system.

2. Formulation

2.1. Displacement formulation for the pile–soil domain

In this paper, we deal with a single, circular pile with diameter B and length L embedded in an elasto-plastic soil deposit and subjected to a lateral load and/or moment at the head. Fig. 1 shows a Cartesian coordinate system that is used to describe the pile–soil domain. The origin of the coordinate system is placed at the center of the pile head, such that the x and y axes are along and perpendicular to the loading direction, respectively, and the z axis points downward along the pile axis.

The pile is modeled as a vertical Euler–Bernoulli beam with pile deflection w in the x-axis direction expressed as a function of depth z:

$$w = w(z) \tag{1}$$

The displacements in the soil result from the pile–soil interaction as the pile is loaded laterally; the greater the pile deflection, the greater the displacement in the soil is. The displacement in the soil at any depth z depends on the position being considered with respect to the pile axis. In general, soil will undergo greater displacements near the pile than far from it. Thus, we can express the soil displacement in each direction as the product of the pile deflection function $w(z)$ and a corresponding decay function:

$$\begin{aligned} u_x &= w(z)f(x,y) \\ u_y &= w(z)g(x,y) \\ u_z &= 0 \end{aligned} \tag{2}$$

where u_x, u_y and u_z are the soil displacements in the x, y, and z directions, and $f(x,y)$ and $g(x,y)$ are the decay functions for the soil displacements in the x–y plane. When a pile is subjected to a lateral load, the displacement in the soil will be mainly in the horizontal direction. The displacement in the vertical direction is negligible when compared to those in the two other directions, and this is why it is disregarded. Differentiation of the displacements gives us the strain components (which are positive in compression):

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix} = -\begin{Bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial u_y}{\partial y} \\ \frac{\partial u_z}{\partial z} \\ \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) \end{Bmatrix} = \begin{Bmatrix} -w(z)\frac{\partial f(x,y)}{\partial x} \\ -w(z)\frac{\partial g(x,y)}{\partial y} \\ 0 \\ -\frac{1}{2}w(z)\left[\frac{\partial f(x,y)}{\partial y} + \frac{\partial g(x,y)}{\partial x}\right] \\ -\frac{1}{2}\frac{dw(z)}{dz}f(x,y) \\ -\frac{1}{2}\frac{dw(z)}{dz}g(x,y) \end{Bmatrix} \tag{3}$$

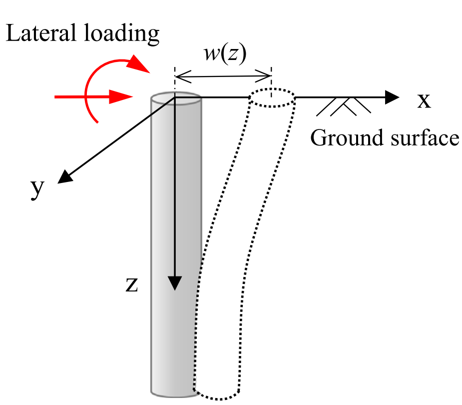

Fig. 1. The Cartesian coordinate system used for the soil–pile domain.

2.2. Principle of virtual work

Energy is input into the pile–soil system when an external lateral force F and a moment M are applied at the pile head, inducing deformation in the pile and the soil surrounding it. The principle of virtual work for the stated problem can be written in the incremental form as:

$$EI\int_0^L \left(\frac{d^2\Delta w}{dz^2}\right)\delta\left(\frac{d^2 w}{dz^2}\right)dz + \int_{\Omega_s}\Delta\sigma_{kl}\delta\varepsilon_{kl}\,d\Omega_s = \Delta F\delta w|_{z=0} - \Delta M\delta\left(\frac{dw}{dz}\right)\Big|_{z=0} \tag{4}$$

where E is the Young's modulus of the pile; I is the second moment of area of the pile's cross section; Δw is the increment in the pile displacement; the stresses σ_{kl} and strains ε_{kl} are written using indicial notation, according to which repetition of any index (such as k or l) in a term implies summation over that index; ΔF and ΔM are the increments in the lateral force and moment applied at the pile head, respectively; and Ω_s denotes the soil domain, which extends from $-\infty$ to ∞ in both the x and y directions and from 0 to ∞ in the z direction except for the volume occupied by the pile. The left hand side of Eq. (4) is the incremental form of the internal virtual work stored in the pile and soil, and the right hand side is the incremental form of the external virtual work done by the external concentrated force and the bending moment. In this incremental formulation of the principle of virtual work, an increment in external forces and internal stresses are used as the equilibrium set, and the current displacements and strains are used as the compatible set. Replacing the incremental terms by the differences between their initial values (denoted by the subscript zero) and final values at each increment, Eq. (4) becomes:

$$\begin{aligned} &EI\int_0^L \left(\frac{d^2 w}{dz^2}\right)\delta\left(\frac{d^2 w}{dz^2}\right)dz + \int_{\Omega_s}\Delta\sigma_{kl}\delta\varepsilon_{kl}\,d\Omega_s - F\delta w|_{z=0} + M\delta\left(\frac{dw}{dz}\right)\Big|_{z=0} \\ &= EI\int_0^L \left(\frac{d^2 w_0}{dz^2}\right)\delta\left(\frac{d^2 w}{dz^2}\right)dz - F_0\delta w|_{z=0} + M_0\delta\left(\frac{dw}{dz}\right)\Big|_{z=0} \end{aligned} \tag{5}$$

2.3. Governing differential equations

2.3.1. Deflection function w(z)

The term $\Delta\sigma_{kl}\delta\varepsilon_{kl}$ in Eq. (5) can be expanded to:

$$\Delta\sigma_{kl}\delta\varepsilon_{kl} = \Delta\sigma_{xx}\delta\varepsilon_{xx} + \Delta\sigma_{yy}\delta\varepsilon_{yy} + 2\Delta\sigma_{xy}\delta\varepsilon_{xy} + 2\Delta\sigma_{xz}\delta\varepsilon_{xz} + 2\Delta\sigma_{yz}\delta\varepsilon_{yz} \tag{6}$$

Since we aim to formulate the differential equations for the pile deflection, all the terms in Eq. (6) need to be expressed in terms of the displacements. This is accomplished by replacing the stress increments $\Delta\sigma$ by the product of the tangential stiffness matrix C_{ij} and the strain increments and writing the strains in terms of the displacements [by using Eq. (3)]. For example, the first term $\Delta\sigma_{xx}\delta\varepsilon_{xx}$ can be expanded as:

$$\begin{aligned} \Delta\sigma_{xx}\delta\varepsilon_{xx} &= [C_{11}\Delta\varepsilon_{xx} + C_{12}\Delta\varepsilon_{yy} + C_{13}\Delta\varepsilon_{zz} + C_{14}\Delta\varepsilon_{xy} + C_{15}\Delta\varepsilon_{xz} + C_{16}\Delta\varepsilon_{yz}]\delta\varepsilon_{xx} \\ &= \left\{ w\frac{\partial f}{\partial x}\left[C_{11}\frac{\partial f}{\partial x} + C_{12}\frac{\partial g}{\partial y}\right]\delta w + ww\left[C_{11}\frac{\partial f}{\partial x} + C_{12}\frac{\partial g}{\partial y}\right]\delta\left(\frac{\partial f}{\partial x}\right) \right. \\ &\quad + \frac{1}{2}C_{14}w\left[\frac{\partial f}{\partial y} + \frac{\partial g}{\partial x}\right]\frac{\partial f}{\partial x}\delta w + \frac{1}{2}C_{14}ww\left[\frac{\partial f}{\partial y} + \frac{\partial g}{\partial x}\right]\delta\left(\frac{\partial f}{\partial x}\right) \\ &\quad \left. + \frac{1}{2}\frac{dw}{dz}\frac{\partial f}{\partial x}[C_{15}f + C_{16}g]\delta w + \frac{1}{2}\frac{dw}{dz}w[C_{15}f + C_{16}g]\delta\left(\frac{\partial f}{\partial x}\right) \right\} \\ &\quad - \left\{ w_0\frac{\partial f}{\partial x}\left[C_{11}\frac{\partial f_0}{\partial x} + C_{12}\frac{\partial g_0}{\partial y}\right]\delta w + w_0 w\left[C_{11}\frac{\partial f_0}{\partial x} + C_{12}\frac{\partial g_0}{\partial y}\right]\delta\left(\frac{\partial f}{\partial x}\right) \right. \\ &\quad + \frac{1}{2}C_{14}w_0\delta w\left[\frac{\partial f_0}{\partial y} + \frac{\partial g_0}{\partial x}\right]\frac{\partial f}{\partial x} + \frac{1}{2}C_{14}w_0 w\left[\frac{\partial f_0}{\partial y} + \frac{\partial g_0}{\partial x}\right]\delta\left(\frac{\partial f}{\partial x}\right) \\ &\quad \left. + \frac{1}{2}\frac{dw_0}{dz}\frac{\partial f}{\partial x}[C_{15}f_0 + C_{16}g_0]\delta w + \frac{1}{2}\frac{dw_0}{dz}w[C_{15}f_0 + C_{16}g_0]\delta\left(\frac{\partial f}{\partial x}\right) \right\} \end{aligned} \tag{7}$$