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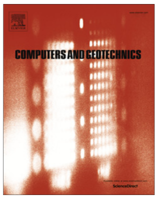

A micromechanical model for the elastic–plastic behavior of porous rocks

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ABSTRACT

In this paper, we propose a polycrystalline model to study the elastic–plastic behavior of porous rocks. The proposed model will be applied to sandstone. For this purpose, the microstructure of porous rocks will be represented by an assembly of discrete grains and pores. The plastic deformation of each grain is related to the frictional sliding along a number of weakness planes. A specific plastic model is devised to describe the sliding phenomena to take into account the characteristics of rocks such as the pressure dependency and the volumetric dilatancy. A homogenization approach–self-consistent is developed to account for the interactions between grains and pores. An efficient numerical procedure for model implementation is proposed. Finally, we present a series of numerical investigations and comparisons with experimental data to assess the capability of the proposed model to capture the main features of mechanical behaviors of porous rocks.

1. Introduction

Porous rocks are frequently encountered in civil engineering applications. Their macroscopic mechanical behaviors and hydraulic properties are directly affected by the porosity. A number of experimental investigations have been performed on various porous rocks, for instance [1–7], just to mention a few. Most experimental results have shown that the mechanical behaviors of porous rocks exhibit some specific features such as strong pressure sensitivity, plastic pores collapse, strong dependency on pore distribution.

Based on experimental evidences, macroscopic elastic–plastic models have first been developed in the framework of irreversible thermodynamics. The particularity of such models for porous rocks is generally the presence of a cap yield surface in order to account for the plastic pore collapse due to hydrostatic compression. This is done either by using two distinct yield surfaces [8,9,6,10] or a closed single yield surface [11,12]. Some studies have been also devoted to poromechanical modelling of saturated porous rocks [9,6,10,13]. However, in these models, the yield functions and plastic potentials do not explicitly depend on the porosity and mineral composition of porous rocks. In order to improve such macroscopic models and in order to properly take into account the effect of porosity, inspired by the reference work by Gurson for porous metal materials [14], some micro–macro upscaling models based on different nonlinear homogenization techniques have been proposed for porous geomaterials, for instance [15–18]. In these models, pores are embedded in a pressure-sensitive solid matrix and the macroscopic yield criteria are explicitly functions of porosity. This kind of approach has even been extended to geomaterials containing other mineral grains inside the porous matrix [19,20]. Some authors have also considered the time-dependent behavior related to viscoplastic flow of porous matrix [21]. However, in these micro–macro models, the microstructure of porous materials is generally represented by a matrix-inclusion system. Pores are embedded as void inclusions inside a solid matrix. Such a particular microstructure is not suitable for a class of porous rocks. According to some microstructural analysis, for instance the scanning electron microscope (SEM) image of sandstone shown in Fig. 1, reported by [5]. The microstructure of sandstone is rather an assembly of mineral grains and pores as well as interfaces between grains. In spite of the microstructural complexity and as a first approximation, such kind of microstructure is quite similar to that of polycrystalline materials. The macroscopic behaviors of porous rocks shall depend on the deformation of mineral grains, the change of porosity and the deformation of interfaces. Further, it is also observed that the grain distribution at the microscopic scale is not fully isotropic in nature. This can be the origin of an

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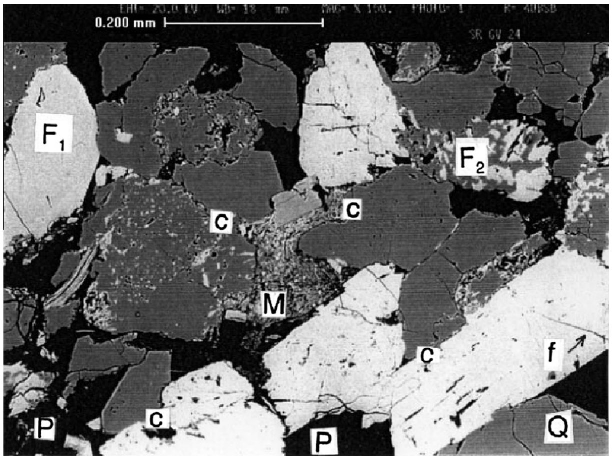

Fig. 1. Scanning electron micrograph of Vosges sandstone [5].

anisotropic behavior of material at the macroscopic scale. However, in most experimental studies performed so far on sandstone, it was generally accepted that the overall mechanical of this class of porous rocks can be reasonably described by an isotropic model.

Therefore, in the present paper, we will propose a micromechanical model for the description of elastic–plastic behavior of porous rocks with a polycrystalline microstructure. Due to the particularity of microstructure, a self-consistent approach is needed in order to properly consider interactions between mineral grains and pores. Our model will be based on the classical polycrystalline models for metal materials but with a suitable adaptation taking into account specific features of porous rocks. Firstly, the crystallographic planes are extended to weakness planes (cracks, flaws, etc.) in various orientations. The plastic sliding along the weakness planes is controlled by the friction property and influenced by the local normal stress. A Mohr–Coulomb type yield criterion is therefore adopted. This corresponds to the macroscopic pressure-dependent plastic behavior. Secondly, the roughness of weakness planes is taken into account. The tangential sliding can produce normal opening of weakness planes. At the macroscopic scale, the deviatoric loading can generate volumetric dilatancy, which is influenced by the mean stress. This kind of behavior is frequently observed in rocks. Finally, the effect of porosity is taken into account. Due to the strong interaction between mineral grains and pore, it is needed to use a fully self-consistent approach with a robust numerical framework for the implementation of the micromechanical model. We will propose an efficient multi-level loop procedure for solving problems with multiple interactive plastic slip systems. However, as the first stage of our research work, in the present paper, we will neglect the deformation and damage on interface between grains. This important feature is still considered in our ongoing work.

The following tensor notations and operations are employed throughout this paper: first order tensor $\underline{a}$; second order tensor $\underline{\underline{a}}$; fourth order tensor $\mathbb{A}$; double contraction $\underline{\underline{a}} : \underline{\underline{b}} = a_{ij}b_{ij}$, $\mathbb{A} : \underline{\underline{b}} = A_{ijkl}b_{kl}$, $\mathbb{A} : \mathbb{B} = A_{ijkl}B_{klmn}$; dyadic products $\underline{\underline{a}} \otimes \underline{\underline{b}} = a_i b_j$, and its symmetrical part $\underline{a} \overset{s}{\otimes} \underline{b} = (a_i b_j + a_j b_i)/2$. Three commonly used fourth order isotropic tensors are expressed as: $I_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$, $K_{ijkl} = \frac{1}{3}\delta_{ij}\delta_{kl}$, $J_{ijkl} = I_{ijkl} - K_{ijkl}$. δ_{ij} is the Kronecker delta tensor.

2. Summary of some experimental results

For the sake of clarity, we consider here a typical sandstone as a reference porous rock. It is the Vosges sandstone, which comes from the Vosges mountains in France. It is a pink quartz sandstone (quartz = 93%), with a few percent of feldspar and white mica. The average porosity is about 20%. As the porosity of Vosges sandstone is relative high, it was determined by simply comparing the weight difference between the dry sample and the saturated one. On the other hand, a typical micrograph of intact sandstone by scanning electron microscopy (SEM) [5], is shown in Fig. 1. We can see that the microstructure of sandstone is basically constituted of various mineral grains, pores and interfaces.

A series of triaxial compression tests with different confining pressures has been conducted to study its mechanical behavior [1,5]. Typical stress–strain curves are shown in Fig. 2. From these results, we can first see that the slope of the linear part is not constant and increases with the confining pressure. By considering this slope as the initial elastic modulus of material, we have calculated the values of elastic modulus and Poisson's ratio for different confining pressures, as shown in Table 1. One can see that the elastic modulus increases with the confining pressure while the Poisson's ratio varies rather in a irregular manner. Among other phenomena, the increase of the elastic modulus can be due to the compaction of pores and then the decrease of porosity by the application of confining pressure. The peak stress also increases with the confining pressure but in a non-linear manner, indicating that the macroscopic failure criterion corresponds to a curved surface, as shown in Fig. 3. The volumetric strain (E_v) is also presented in Fig. 2. One can see that the volumetric strain rate is first compressive and then progressively becomes dilatant when the deviatoric stress increases. There is a transition from the volumetric compressibility to dilatancy in terms of volumetric strain rate. This phenomenon is a typical behavior of a large class of rock like materials.

3. Formulation of the micromechanical model

As stated above, the unit cell of sandstone is an assembly of quartz grains, pores and interfaces. At this stage, the deformation of interfaces is neglected. When the unit cell is subjected to a uniform macroscopic strain at its remote boundary, non-uniform stress and strain fields are generated in each grain. The relationship between the macroscopic strain and the local fields depends on the interaction between grains and pores. The description of this relationship is the key point of homogenization procedure. As there is no dominant material phase in the unit cell, each grain is embedded in a unknown matrix which is the homogenized equivalent

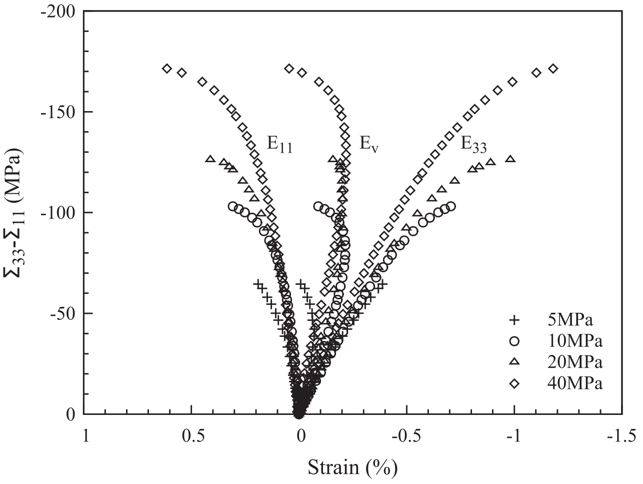

Fig. 2. Typical stress–strain curves of Vosges sandstone under different confining pressures [1].