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Research Paper

Slope stability analysis by means of finite element limit analysis and finite element strength reduction techniques. Part I: Numerical studies considering non-associated plasticity

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ABSTRACT

Slope stability analyses in practical geotechnical engineering are predominantly performed using limit equilibrium methods, despite the inherent shortcoming that the form of the failure mechanism has to be defined *a priori*. This assumption is not needed when more advanced methods, such as limit analyses or displacement-based finite element methods, are employed for calculating factors of safety and thus the advantages of these methods are increasingly recognized. However, the latter may suffer from numerical instabilities when using non-associated plasticity whereas the former are restricted to associated flow rules. This paper shows that these issues may be overcome by a modification of the so-called Davis approach which provides accurate estimates of the factor of safety of slopes, even for extreme cases of steep slopes with friction angles in excess of 40° and zero dilatancy.

1. Introduction

This paper compares the results obtained from finite element limit analysis (FELA) and displacement finite element strength reduction techniques (SRFEA) for slope stability problems. It has been shown previously that both approaches yield almost exactly the same results when employing a Mohr–Coulomb criterion with an associated flow rule [19], but for non-associated plasticity numerical instabilities may occur with displacement-based finite element methods. This is especially the case for problems with a high degree of non-associativity, where the friction angle is much greater than the dilation angle and the so-called Davis approach has to be used in finite element limit analyses (because these methods can only deal with associated plasticity). Although the Davis [3] approach works reasonably well when the factors of safety are based on optimizing a load vector for a given strength, it may yield (very) conservative results when the FoS is based on the strength of the soil. In order to overcome this limitation, a modification of the original approach of Davis [3] is suggested. Results show that similar factors of safety are obtained with FELA and SRFEA when solving two problems. Application to a case history, employing the new approach, will be presented in a companion paper.

2. Numerical methods used for comparison of factors of safety

2.1. Strength reduction method with displacement finite element method (SRFEA)

With displacement finite element formulations a factor of safety (FoS) against failure in a soil mass can be obtained by means of the strength reduction method (SRFEA). This is performed with characteristic strength properties for the friction angle φ' and the cohesion c', followed by an incremental decrease of tan φ' and c' (assuming a Mohr–Coulomb failure criterion). This leads to stress states that violate the strength criterion, leading to a stress redistribution in the system until equilibrium can no longer be established and failure is reached. However, close inspection of the failure mechanism developed, as well as the displacements of appropriate control points, is required in order to avoid misinterpretation. The factor of safety obtained from the procedure is defined by:

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Nomenclature

c'	effective cohesion
$c'_{mob.}$	mobilized effective cohesion during SRFEA
c^*	reduced cohesion according to Davis [3]
H_s	slope height
α_s	slope angle
β	strength factor according to Davis [3]
β_0	strength factor according to Davis [3] at initial conditions
$\beta_{failure}$	strength factor according to Davis [3] at failure
γ	unit weight
φ'	effective friction angle
$\varphi'_{failure}$	effective friction angle at failure
$\varphi'_{mob.}$	mobilized effective friction angle during SRFEA
φ^*	reduced friction angle according to Davis [3]
η	inclination to vertical direction
$\delta\varepsilon_1$	major principal strain increment
$\delta\varepsilon_3$	minor principal strain increment
$\delta\varepsilon_n$	zero direct strain increment
$\delta\varepsilon_{vol}$	volumetric strain increment
$\delta\gamma$	engineering shear strain increment
$\delta\gamma_{max}$	maximum engineering shear strain increment
ψ'	dilatancy angle
$\psi'_{failure}$	dilatancy angle at failure
FELA	finite element limit analysis
FoS	factor of safety
FoS_{UB}	factor of safety obtained with upper bound analysis
FoS_{LB}	factor of safety obtained with lower bound analysis
P_P	pole point for planes
Λ	amount of non-associativity $(\varphi' - \psi')$
σ'_k	effective normal stress based on velocity characteristics
σ'_s	effective normal stress which defines failure criterion according to Coulomb
SRFEA	strength reduction finite element analysis
τ_k	shear stress based on velocity characteristics
τ_s	shear stress which defines failure criterion according to Coulomb
σ'_1	major effective principal stress
σ'_3	minor effective principal stress

$$\mathrm{FoS} = \frac{\tan\varphi'}{\tan\varphi'_{\mathrm{mobilized}}} = \frac{c'}{c'_{\mathrm{mobilized}}} \tag{1}$$

For strength reduction with more complex models where the strength is a function of state variables, a more sophisticated algorithm is required (see Potts and Zdravkovic [12]).

2.2. Factor of safety obtained from finite element limit analysis (FELA)

Finite element formulations of the upper- and lower-bound theorems of plasticity have developed markedly over the last two decades, and it is now possible to apply them to a wide variety of complex engineering problems [18].

Finite element limit analysis (FELA) is particularly powerful when both upper- and lower-bound estimates are calculated so that the true collapse load (for the ideal material) is bracketed from above and below. The difference between the two bounds then provides an exact measure of the error in the solution, and can be used to refine the meshes until a suitably accurate estimate of the collapse load is found. The formulations used in this paper stem from the methods originally developed by Sloan [15], Sloan [16] and Sloan and Kleeman [17], and further improved by Lyamin and Sloan [9], Lyamin and Sloan [10] and Krabbenhoft et al. [6], Krabbenhoft et al. [7]. A detailed description of the formulation of the finite element limit analysis methods used in this paper, including the process for adaptive mesh refinement, is given in Sloan [18].

If the safety factor obtained with FELA needs to be expressed in terms of the material strength, which is defined as the ratio between the actual material strength and the mobilized material strength at failure, a strength reduction process must be performed as described in Sloan [18].

3. Numerical studies

3.1. Influence of flow rule in SRFEA

An important issue in displacement finite element analysis of failure is the definition of the flow rule. In SRFEA the flow rule in the automatic φ'/c' reduction procedure, as implemented in the FE-code Plaxis [1] which is employed in this study, is handled as follows: for associated plasticity the dilatancy angle ψ' is reduced incrementally in the same way as the friction angle φ', while for the non-associated case with $\psi' < \varphi'$, ψ' is kept constant as long as the reduced value for φ' is larger than ψ'. Once φ' falls to the value of ψ', both angles are then reduced simultaneously in subsequent iterations. In many non-associated cases, a flow rule with a dilatancy angle ψ' much smaller than the friction angle φ' is employed, but this may lead to numerical instability with no clear indication of the failure mechanism (e.g. Nordal [11]). This issue has also been studied extensively by Krabbenhoft et al. [4,8]. The main problem with a non-associated flow rule is that the solution of the governing equations is not unique. This effect was also investigated by Rice [13], who showed that the non-uniqueness of the solution is related to the occurrence of shear bands (bifurcation). As a consequence of this bifurcation the computed limit load with a non-associated flow rule is smaller compared to the one obtained with associated plasticity. In addition, the flow rule influences the kinematics of the failure mechanism, which, compared to analysis with associated plasticity, leads to a reduction of the computed FoS. It is emphasized that the problem of non-associated flow in slope stability analysis is not significant for friction angles up to approximately 35°. For these cases, the flow rule has a minor influence on the calculated factor of safety, which are similar to those obtained with limit equilibrium methods (see e.g. Cheng et al. [2]).

To study this issue a simple FE model of a steep slope (height H_s = 10.0 m and inclination α_s = 45°) in a homogeneous soil layer with a high friction angle is considered. Fig. 1a shows the model dimensions and the mesh discretization with 15-noded triangular elements. The analyses consider drained conditions and a linear elastic – perfectly plastic constitutive model with a Mohr–Coulomb failure criterion. The effective friction angle φ' is 45°, the effective cohesion c' is 6.0 kPa and the unit weight γ is 20.0 kN/m^3. To investigate the effect of the dilatancy angle five different values for ψ' are used; namely 0, 5°, 10°, 15° and 45°.

In the first calculation phase gravity loading is applied and subsequently a SRFEA is performed. Fig. 1b shows the evaluation of FoS over calculation steps for the cases with associated and non-associated flow rules. One can see that the flow rule does have a significant influence on the factor of safety and, as expected, the associated case yields the highest value of about 1.53 which